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Second law thermodynamic study of heat exchangers: A review

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ABSTRACT

Heat exchangers are thermal systems which are used extensively, have a major role in energy conservation aspect and preventing global warming. This paper is based on reviews of scientific work and provides a state-of-the-art review of second law of thermodynamic analysis of heat exchangers. Initially, the basics of heat exchangers are briefly provided along with second law analysis which also includes two-phase flow analysis and thermoeconomic analysis. Following this, detail literature survey based on performance parameters such as entropy generation, exergy analysis, production and manufacturing irreversibilities (cumulative exergy destruction associated with the production of material and manufacturing of component or assembly of components) and two phase fluid loss of heat exchangers is presented including constructal law applied to analyze heat exchangers. Constructal theory along with second law analysis can be used for the systematic design of heat exchangers. This review highlights the importance of first and second law investigations of heat exchangers leading to energy conservation.

Contents

1. Introduction

Energy conservation is key goal of the world economy and will continue to be one in the future. The most effective way to reduce energy demand is to use energy more efficiently. Heat exchangers are widely used in power engineering, chemical industries, petroleum refineries, food industries and in *HVAC* (heating, ventilating and air conditioning) technology. Therefore, heat transfer and the design of heat transfer equipment continue to be a centrally important issue in energy conservation. The optimal use of energy and efficient heat transfer has become a vital importance as a result of the diminishing world energy resources and increasing energy cost. Therefore, the number of investigations on heat transfer enhancement has progressively increased. As a result, it is very important to determine the performance of

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Nomenclature

A	surface area, m^2
Be	Bejan number
C	heat capcity rate, W/K
c	specific heat, J/kg K
c_p	specific heat at constant pressure, J/kg K
c_v	specific heat at constant volume, J/kg K
C_r	capacity ratio
D	diameter, m
$\dot{E}$	exergy rate, W
E_{vh}	entransy, J K
h	specific enthalpy, J/kg
$EEIN$	enthalpy exchange irreversibility norm
$HVAC$	heating, ventilating and air conditioning
$\dot{I}$	irreversibility, W
k	thermal conductivity, W/m K
L	length, m
$\dot{m}$	mass flow rate, kg/s
M	mass, kg
n	number of constructal level
Ntu	number of heat transfer units
P	pressure, Pa
ΔP	pressure drop, Pa
Q	heat transfer, J
$\dot{Q}$	heat transfer rate, W
R	gas constant, J/kg K
s	specific entropy, J/kg K
S	entropy, J/K
$\dot{S}_{gen}$	entropy generation rate, W/K
Sv	svelteness
T	temperature, K
ΔT	temperature difference, K
T_o	reference temperature, K
u	specific internal energy, J/kg
U	overall heat transfer coefficient, W/m^2 K
$\dot{W}$	Work, W
Y_S	Heat Exchange Reversibility Norm (HERN)

Greek symbols

ε	effectiveness
ρ	density, kg/ m^3
υ	kinematic viscosity, m^2/s
ψ	rational efficiency

Subscripts

1	stream 1
2	stream 2
c	cold stream
h	hot stream
H	heat transfer term
in	inlet
m	material
max	maximum
min	minimum
out	outlet
P	pressure drop term
sat	saturated condition
tp	two-phase
v	vapor

heat exchange devices on both heat transfer and thermodynamic considerations. Heat exchangers are the equipments that provide the flow of thermal energy between two or more fluids at different temperatures. Heat exchangers are used in variety of applications. There are four basic types of losses that occur in a typical heat exchanger as referred from Bejan [1–6]:

(i) losses due to the exchange of heat across a finite temperature difference,
(ii) losses due to fluid friction,
(iii) losses due to material and manufacturing of heat exchanger [7], and
(iv) thermal losses due to heat exchange with the environment.

The last heat exchanger losses are usually small, because the heat exchanger surface is insulated to reduce such an exchange of heat. Other losses are evaluated using thermodynamic analysis of heat exchangers. The first law of thermodynamics deals with the quantitative conservation of energy in various forms transferred between the system and its surroundings and with the changes in the energy stored in the system. It treats work and heat interactions as equivalent form of energy in transit. It deals with the quantity of energy and asserts that energy cannot be created or destroyed. The second law however, deals with the quality of energy. More specifically, it is concerned with the degradation of energy during a process, the entropy generation and lost opportunities to do work. The second law of thermodynamics has proved to be a very powerful tool in the optimization of complex thermodynamic systems and is required to establish the difference in quality between mechanical and thermal energy [8].

A reversible process is defined as a process that can be reversed without leaving any trace on the surroundings. Processes that are not reversible are called irreversible processes. The factors that cause a process to be irreversible are called irreversibilities. They include heat transfer across a finite temperature difference, friction, unrestrained expansion, mixing of fluids, etc. The irreversiblities occurring during a process is called process irreversibility. Entropy is defined as a system property by a statement that its change in an ideal, reversible process must be equal to the transfer of an entity $\int dQ/T$ that accompanies any heat transfer dQ across the system boundary where the local temperature is T. Hence, this abstract system property indicates that heat transfer must be accompanied by an entropy change. As a consequence, a reversible adiabatic process can be identified by zero entropy change. If a process is not reversible (as with any heat transfer across a finite temperature difference), the situation is radically different. Entropy change ΔS is either equal (reversible process) or larger (irreversible process) than the entropy transfer ($\int dQ/T$) that accompanies heat transfer dQ, the difference being attributed to entropy generation $\dot{S}_{gen}$. The amount of entropy generation is the quantitative measure of the quality level of energy transfer. Entropy generation of zero corresponds to the highest quality of energy transfer and/or energy conversion (a reversible process), and entropy generation greater than zero represents poorer quality. All real processes are characterized by entropy generation greater than zero. Irreversibility can be expressed in energy terms as a product of entropy generation and a reference temperature, T_o (i.e., $\dot{I} = T_o\dot{S}_{gen}$).

Heat exchangers are generally inefficient from an energy conservation point of view because they have been designed in

the past on the basis of low cost that dictates a minimum-size unit. To achieve the small-sized heat exchanger, the temperature difference between the fluid streams is maximized. However, the larger is the temperature difference in a heat exchanger, the greater will be the loss during heat transfer. Temperature and temperature difference distributions within the heat exchanger influence thermodynamic irreversibility. The entropy generation is related to the difference in the fluid temperatures of two streams of heat exchanger. Hence, the difference between mean temperatures of two fluids directly influences the entropy measure of the irreversibility manifested within the heat exchanger. As a consequence, the heat exchanger irreversibility for a given heat transfer rate can be reduced by reducing temperature differences between the fluids, which in turn will increase the exchanger effectiveness. A heat exchanger characterized by smaller temperature differences between the fluids generates a smaller irreversibility in a given system compared to a heat exchanger (for the same heat transfer rate) that has larger temperature differences between the fluids.

Also capacity mismatch is used between the streams to increase the performance. Due to which more loss occurs, this cannot be easily known from the first law analysis. Similarly pumping pressure of fluid streams is increased to achieve higher performance. This leads to increase in pressure drop irreversibility. When fluid flows through heat exchanger passage, it is subjected to fluid friction and other pressure drop contributions along the fluid flow path. This leads to pressure drop in fluid which is proportional to pumping power. The irreversibility caused by fluid pressure drop is known as pressure drop irreversibility. That is why the thermodynamic analysis of heat exchangers is an important issue, and heat exchangers should be evaluated using the first law as well as second-law based performance evaluation criteria. The losses due to process irreversibility can be calculated only by using second law analysis.

The irreversibilities associated with the use of materials in a system are known as production and manufacturing irreversibilities. It is the irreversibilities due to the production of the materials and the construction of a component or assembly of components. Also, after the life of the component, it is recycled; those irreversibilities can also be included. It can be calculated by cumulative exergy of a component production process considering cumulative exergy analysis. The cumulative exergy includes the cumulative exergy of raw materials, the electricity consumption, and the contribution of the equipment used in the production process. Because the production of heat exchangers involves mainly machining processes, the energy consumption is relatively small. As a result, the cumulative exergy of raw materials is the major component of the total value. For example, the cumulative exergy destruction associated with the production of material and manufacturing of tubes consists of exergy destruction occurring for obtaining the raw material from ore form, primary processes like casting, extruding, rolling, etc., secondary processes like bending, welding and final manufacturing process like assembling. So, in the second law of thermodynamic analysis of heat exchangers not only the irreversibilities associated with the heat transfer and pressure drop is considered but also, irreversibility associated with the production and manufacturing of material is also considered. This further provides extending the investigations leading to life cycle analysis.

There exists a direct proportionality between irreversibility, quantity of entropy generated and the amount of available work lost in the process. Second law analysis seeks to minimize these losses by keeping the entropy generated to a minimum. A realistic design approach for systems is to base the design on minimum entropy production. In other words, entropy generation can be used to determine quantitatively the quality of thermal energy transformation. Therefore, in the analysis and design of heat exchangers, it is essential to give due consideration to the rate of irreversible entropy generation as well as available energy or exergy destruction process. The performance of energy systems is degraded by the presence of irreversibilities and entropy generation is measure of the magnitude of irreversibilities present during that process. The greater the extent of irreversibilities, the greater will be entropy generation. Therefore, entropy generation can be used as a quantitative measure of irreversibilities associated with a process. So, we need to keep this inevitable entropy generation during a process minimum possible while designing a system to increase the performance.

By constructal theory we can also identify the geometric configuration that maximizes performance subject to several global constraints [4]. Maximum thermodynamic performance is achieved by minimization of the entropy generated in the assemblies. Tree networks represent a new trend in the optimization and miniaturization of heat transfer devices. Constructal theory is used to optimize the performance of thermo-fluid flow systems by generating geometry and flow structure, and to explain natural self-organization and self-optimization. Bejan [4] stated the constructal law as “For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it”. The optimal structure is constructed by optimizing volume shape at every length scale, in a hierarchical sequence that begins with the smallest building block, and proceeds towards larger building blocks (which are called ‘constructs’) [6]. The present review work is an attempt to study heat exchanger design based on second law of thermodynamics and constructal theory.

2. Thermodynamic analysis of heat exchangers

In this section, we are presenting some of the basic concepts of thermal design of heat exchanger from first and second law of thermodynamic analysis. The effectiveness-*Ntu* (number of heat transfer units) method offers many advantages for the analysis of problems in which a comparison between various types of heat exchangers need to be carried out and for selecting the best type suited to accomplish a particular heat transfer objective. The effectiveness-*Ntu* method does not require outlet temperature of the streams. Only the inlet temperature of streams is enough to determine the heat transfer rate of the heat exchanger. If outlet temperatures are known, then effectiveness-*Ntu* can be used for heat exchanger sizing problem. The heat exchanger effectiveness is defined as,

Effectiveness = Actual heat transfer/Maximum possible heat transfer = $\varepsilon = \dot{Q}/\dot{Q}_{max}$.

The actual heat transfer may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid. For heat exchangers it is given as,

$$\dot{Q} = (\dot{m}c_P)_h(T_{h,in} - T_{h,out}) = (\dot{m}c_P)_c(T_{c,out} - T_{c,in}) \tag{1}$$

The maximum possible heat transfer is expressed as,

$$\dot{Q}_{max} = (\dot{m}c_P)_{min}(T_{h,in} - T_{c,in}) \tag{2}$$

The minimum fluid may be either the hot or cold fluid, depending on the mass flow rates and specific heats. The effectiveness equation is given as,

$$\varepsilon = \frac{(\dot{m}c_P)_c(T_{c,out} - T_{c,in})}{(\dot{m}c_P)_{min}(T_{h,in} - T_{c,in})} = \frac{(\dot{m}c_P)_h(T_{h,in} - T_{h,out})}{(\dot{m}c_P)_{min}(T_{h,in} - T_{c,in})} \tag{3}$$

For the given effectiveness and the maximum heat transfer rate, the actual heat transfer rate can be obtained as,

$$\dot{Q} = \varepsilon(\dot{m}c_p)_{min}(T_{h,in} - T_{c,in}) \tag{4}$$

The number of heat transfer units which designates the non dimensional heat transfer size of the heat exchangers is defined as,

$$Ntu = \frac{UA}{C_{min}} \tag{5}$$

Defining capacity rate as the product of mass flow rate and specific heat as,

$$(\dot{m}c_p)_c = C_c \text{ and } (\dot{m}c_p)_h = C_h \tag{6}$$

Accordingly, C_{min} and C_{max} will be minimum and maximum capacity rate respectively. The relationship between effectiveness and number of heat transfer units, Ntu is given for different types of heat exchanger configurations referred from Kays and London [9] as follows.

(a) Counter flow heat exchanger:

$$\varepsilon = \frac{1 - e^{-Ntu(1 - C_{min}/C_{max})}}{1 - (C_{min}/C_{max})e^{-Ntu(1 - C_{min}/C_{max})}} \tag{7}$$

(b) Parallel flow heat exchanger:

$$\varepsilon = \frac{1 - e^{-Ntu(1 + C_{min}/C_{max})}}{1 + (C_{min}/C_{max})} \tag{8}$$

(c) Cross flow, one fluid mixed, other unmixed heat exchanger:

$(C_{max} = C_{mixed}, \quad C_{min} = C_{unmixed})$

$$\varepsilon = \frac{C_{max}}{C_{min}}\left(1 - e^{-(1 - e^{-Ntu})C_{min}/C_{max}}\right) \tag{9}$$

(d) Cross flow, both fluids mixed:

$$\varepsilon = \frac{Ntu}{Ntu/1 - e^{-Ntu} + (C_{min}/C_{max})Ntu/1 - e^{-Ntu(C_{min}/C_{max})} - 1} \tag{10}$$

(e) For condenser and evaporator

$$\varepsilon = 1 - e^{-Ntu} \tag{11}$$

2.1. Second law of thermodynamics analysis

Following the procedure provided in Bejan [10], the degree of thermodynamic imperfection of the arrangement is measured by the entropy generation rate as,

$$\dot{S}_{gen} = \dot{m}_c(s_{c,out} - s_{c,in}) + \dot{m}_h(s_{h,in} - s_{h,out}) \tag{12}$$

Expressing the ideal gas entropy changes in terms of the end pressures and temperatures, Eq. (12) becomes,

$$\dot{S}_{gen} = (\dot{m}c_p)_c \ln\left(\frac{T_{c,out}}{T_{c,in}}\right) + (\dot{m}c_p)_h \ln\left(\frac{T_{h,out}}{T_{h,in}}\right) - (\dot{m}R)_c \ln\left(\frac{p_{c,out}}{p_{c,in}}\right) - (\dot{m}R)_h \ln\left(\frac{p_{h,out}}{p_{h,in}}\right) \tag{13}$$

The first two terms in the above equation represents the heat transfer entropy generation rate while third and fourth terms represents the pressure drop entropy generation rate which we can represent symbolically as,

$$\dot{S}_{gen} = (\dot{S}_{gen})_{\Delta T} + (\dot{S}_{gen})_{\Delta P} \tag{14}$$

In Eq. (13), if the working fluid is liquid (incompressible fluid) in both the streams, the entropy generation rate equation is given as,

$$\dot{S}_{gen} = (\dot{m}c_p)_c \ln\left(\frac{T_{c,out}}{T_{c,in}}\right) + (\dot{m}c_p)_h \ln\left(\frac{T_{h,out}}{T_{h,in}}\right) + \left(\frac{\dot{m}}{\rho}\right)_c (\Delta P)_c + \left(\frac{\dot{m}}{\rho}\right)_h (\Delta P)_h \tag{15}$$

where

$\Delta P = P_{in} - P_{out}$

In Eq. (13), if the working fluid is liquid (incompressible fluid) in both the streams and under non-adiabatic conditions, the entropy generation rate equation is given as,

$$\dot{S}_{gen} = (\dot{m}c_p)_c \ln\left(\frac{T_{c,out}}{T_{c,in}}\right) + (\dot{m}c_p)_h \ln\left(\frac{T_{h,out}}{T_{h,in}}\right) + \left(\frac{\dot{m}}{\rho}\right)_c (\Delta P)_c \frac{\ln(T_o/T_{c,in})}{(T_o - T_{c,in})} + \left(\frac{\dot{m}}{\rho}\right)_h (\Delta P)_h \frac{\ln(T_o/T_{h,in})}{(T_o - T_{h,in})} \tag{16}$$

Defining entropy generation number by dividing entropy generation by minimum heat capacity rate i.e. C_{min} (Bejan [10]),

$$N_S = \frac{\dot{S}_{gen}}{C_{min}} \tag{17}$$

Considering C_{max}=C_h and C_{min}=C_c and expressing outlet temperature in terms of effectiveness from Eq. (3), the total entropy generation number from Eq. (13) for heat exchanger becomes,

$$N_S = \ln\left[1 + \varepsilon\left(\frac{T_{h,in}}{T_{c,in}} - 1\right)\right] + \frac{C_{max}}{C_{min}} \ln\left[1 - \frac{C_{min}}{C_{max}}\varepsilon\left(1 - \frac{T_{c,in}}{T_{h,in}}\right)\right] - \frac{1}{C_{min}}\left[(\dot{m}R)_c \ln\left(\frac{p_{c,out}}{p_{c,in}}\right) + (\dot{m}R)_h \ln\left(\frac{p_{h,out}}{p_{h,in}}\right)\right] \tag{18}$$

The first two terms represents the entropy generation number due to heat transfer and the last two terms represents the entropy generation number due to pressure drop. This can be represented symbolically as,

$$N_S = N_{S,\Delta T} + N_{S,\Delta P} \tag{19}$$

The N_s expression of Eq. (18) is valid for a heat exchanger with any flow arrangement by employing its appropriate ε-Ntu expression viz., counter flow, parallel flow, cross flow, cross flow mixed, cross flow unmixed etc.

The maximum useful work that could be obtained from the system at a given state in a specified environment is known as exergy also called available energy. The property exergy serves as a valuable tool in determining the quality of energy and comparing work potentials of different energy sources or systems. The exergy change of the hot and cold streams can be written with help of the ideal gas relations as follows [8]:

$$\dot{E}_{out} - \dot{E}_{in} = \dot{m}[h_{out} - h_{in} + T_o(s_{out} - s_{in})] = \dot{m}c_P(T_{out} - T_{in}) - T_o\dot{m}c_P \ln(T_{out}/T_{in}) + \dot{m}T_oR \ln(P_{out}/P_{in}) \tag{20}$$

The pressure drop term in Eq. (20) is derived assuming the working fluid as ideal gas. If the fluid is liquid (incompressible fluid), the pressure drop term takes the following form as,

$$\dot{E}_{out} - \dot{E}_{in} = \dot{m}[h_{out} - h_{in} + T_o(s_{out} - s_{in})] = \dot{m}c_P(T_{out} - T_{in}) - T_o\dot{m}c_P \ln(T_{out}/T_{in}) + \dot{m}(P_{out} - P_{in})/\rho \tag{21}$$

The irreversibility, also called exergy destruction or exergy loss, is calculated by exergy balance and taking the difference between all incoming and outgoing exergy flows given by Kotas [11],

$$\dot{I} = \sum\dot{E}_{in} - \sum\dot{E}_{out} \tag{22}$$

Another way of calculating the irreversibility can be done by the Gouy–Stodola formula, in which the entropy generation rate is multiplied by the environmental temperature as,

$$\dot{I} = T_o\dot{S}_{gen} \tag{23}$$

Which in turn can be written in terms of summation of irreversibilities due to temperature difference between the fluid streams and pressure drop respectively as,

$$\dot{I} = \dot{I}_{\Delta T} + \dot{I}_{\Delta P} \tag{24}$$

With the help of entropy generation equations due to heat transfer and pressure drop, the total irreversibility can be written as,

$$\dot{I} = T_o[(\dot{S}_{gen})_{\Delta T} + (\dot{S}_{gen})_{\Delta P}] \tag{25}$$

The irreversibility due to manufacturing of heat exchanger can also be included in the total irreversibility term of Eq. (25) which is provided by Aceves-Saborio et al. [7] and Cornelissen and Hirs [12] as,

$$\dot{I}_m = \frac{ME_d}{t} \tag{26}$$

where M is the mass of heat exchanger which can be calculated by volume and density of the material, E_d is the cumulative exergy destruction of the material and t is the operating time.

2.2. Second law analysis of two-phase heat exchanger

By using first and second laws for the two phase heat exchanger, the entropy generation rate equation is given by Bejan [2] as,

$$\dot{S}_{gen,tp} = \frac{\dot{Q}_{tp}\Delta T}{T_{sat}T_o} + \frac{\dot{m}_{tp}\Delta P_{tp}}{T_{sat}\rho_V} \tag{27}$$

The first term is the entropy generation due to heat transfer and the second term is entropy generation due to pressure drop in two phase fluid, where T_o is the reference temperature.

2.3. Thermoeconomic analysis of heat exchanger

Optimization of thermal systems is generally based on thermodynamic analysis. However, the systems so optimized are often not viable due to economic constraints. The thermoeconomic concept based simplified cost minimization methodology calculates the economic costs of all the internal flows and products of the system by formulating thermoeconomic cost balances. The total cost of the heat exchanger is given as the sum of the capital cost and the irreversibility penalty costs as provided by Bejan et al. [13] and Aceves-Saborio et al. [7] as,

$$C = C_e R_f \varphi + C_s n_h T_o(\dot{S}_{gen,H} + \dot{S}_{gen,P}) \tag{28}$$

where C_e is cost of the equipment which is proportional to surface area of the heat exchanger which includes material and production cost. R_f is the capital recovery factor given as,

$$R_f = \frac{i_e(1+i_e)^{TL}}{(1+i_e)^{TL}-1} \tag{29}$$

where i_e is the effective rate of return, TL is the technical life or life cycle in years and Φ is the operation maintenance factor. C_s is the cost associated with the irreversibilities and n_h is the number of operation hours in a year.

3. Literature review

The literature review has been classified into five subsections to understand the applications of second law analysis on heat exchanger thermal design. Many investigations on heat exchanger design and performance are available in the literature based on second law analysis in which (i) entropy generation and (ii) exergy is used as an evaluation parameter [14]. Further, the literature survey regarding (iii) inclusion of production irreversibility term, (iv) two phase heat exchanger analysis and (v) constructal heat exchangers and analysis have also been reviewed from the point of view of identifying gaps in the literature on thermodynamic aspects of heat exchangers. It is evident that in the forth coming years, the investigations of thermal design of heat exchangers based on entropy and exergy as performance parameters will increase tremendously.

3.1. Entropy generation as performance parameter

Entropy analysis has been carried out in different ways by the various investigators. Initial work on entropy generation in heat exchangers was done by Bejan [10]. Entropy generation rate is contributed by the heat transfer and fluid friction. The most common way of non-dimensionalizing entropy generation rate is to divide it by the capacity flow rate as shown by Bejan [10], which is called as entropy generation number N_s as defined in Eq. (18). This number is second-law "relative" of the older heat transfer concept of the number of heat transfer units (Ntu) (as defined in Eq. (5)), which is used in traditional first-law analyses of heat exchangers. In a similar manner to the Ntu, the value of this entropy generation number can vary in the range of "0 to ∞". Higher values of the entropy generation number correspond to the conditions of higher losses due to heat transfer irreversibility and fluid friction irreversibility. Bejan [10] derived entropy generation numbers for balanced counter flow heat exchangers; heat exchangers with negligible pressure drop irreversibility, and heat exchangers with remanent (flow imbalance) irreversibilities. Entropy generation in a balanced counter flow heat exchanger with zero pressure drop irreversibility is shown in Fig. 1. It exhibits paradoxical conclusion that the irreversibility of such heat exchangers decreases both as ε tends to 1 (expected) and as ε tends to 0 (unexpected). The entropy generation number, N_s is zero at both $\varepsilon=0$ and $\varepsilon=1$ and its maximum is situated exactly at $\varepsilon=0.5$ [10].

Bejan [3] provided a generalized structure of heat exchanger irreversibility as a sum of three entropy generation numbers as follows:

$$N_S = N_{S,imbalance} + N_{S,\Delta T} + N_{S,\Delta P} \tag{30}$$

The first term on the right hand side represents the remanent irreversibility. The second term on the right hand side represents the heat transfer irreversibility and the third term on the right hand side represents the fluid flow irreversibility (refer to Eq. (18)). The basic structure of total entropy generation rate of a heat exchanger is shown in Fig. 2 as provided in Bejan [10]. Three irreversibilities contribute in a more complicated way to the eventual size of the overall N_s and there is tradeoff between the

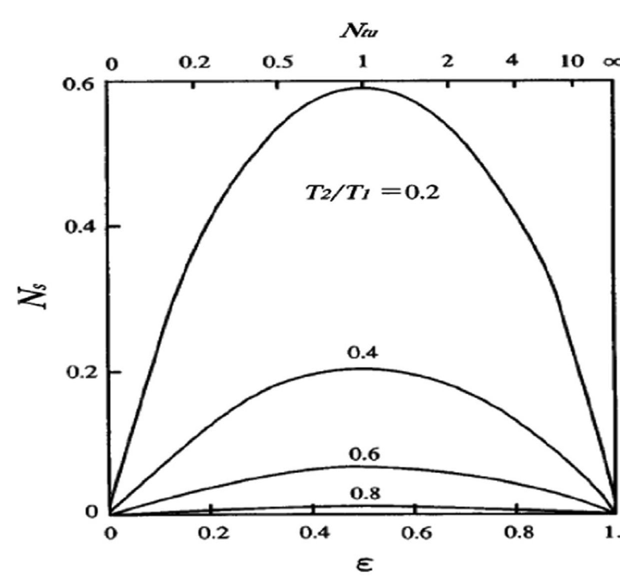

Fig. 1. Entropy generation in a balanced counter flow heat exchanger with zero pressure drop irreversibility [10].

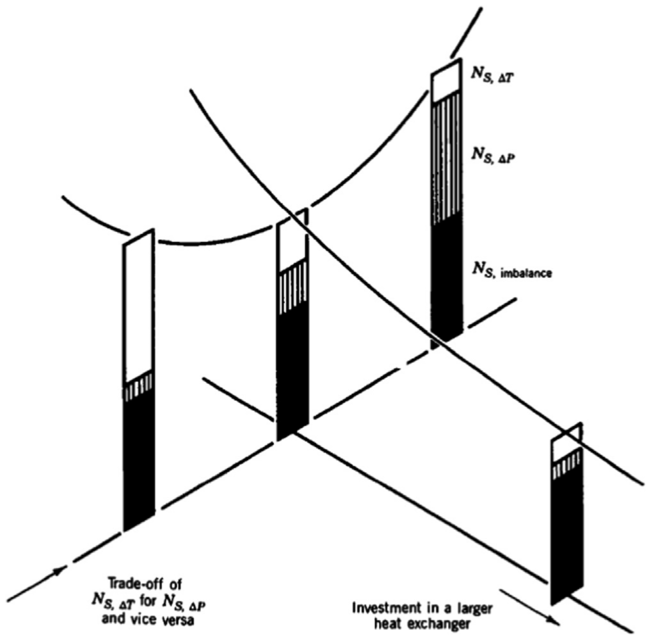

Fig. 2. Structure of entropy generation in a heat exchanger [10].

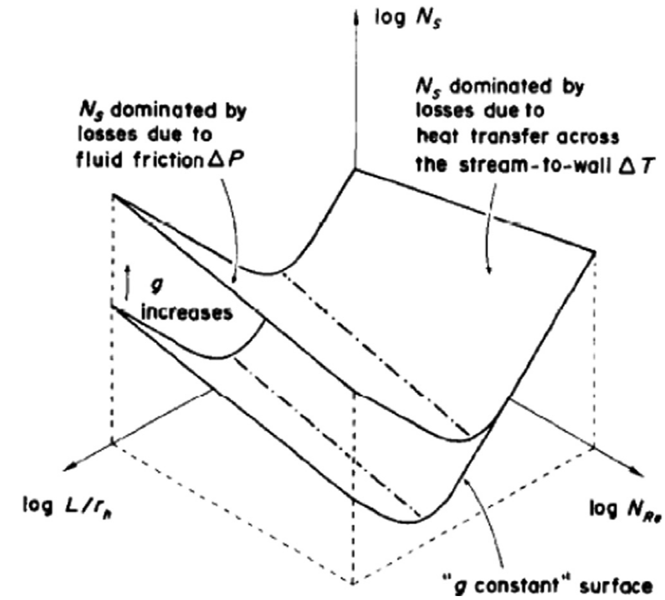

Fig. 3. Schematic diagram of the entropy generation rate for one side of a counter flow heat exchanger, where N_{Re} is the Reynolds number, L and r_h is the length and hydraulic radius of tube and g is the dimensionless mass velocity [10].

irreversibilities. Deep down, however, the behavior of the three is the same as that of the simple limits singled out.

They suggested to calculate the remanent irreversibility ($N_{s,imbalance}$) first in the thermodynamic optimization of any heat exchanger because it is not logic to invest heat exchanger area and "engineering" into minimizing the sum ($N_{S,\Delta T}+N_{S,\Delta P}$) when this sum is already negligible compared with the remanent irreversibility ($N_{s,imbalance}$). Only in very special case does the entropy generation rate of a heat exchanger break into a sum of these three terms. Behavior of the entropy generation rate for one side of a counter flow heat exchanger is shown in Fig. 3. The optimum L/r_h as well as other features of the entropy generation is shown schematically. The minimum rate of entropy generation depends mainly on the mass velocity g, despite a weak dependence on Reynolds number. Therefore, as soon as the mass velocity is fixed, the designer can estimate the theoretically minimum rate of entropy generation for that side of the apparatus.

One such case is the balanced counter flow heat exchanger in the nearly balanced and nearly ideal limit ($C\to 1$, $\Delta T\to 0$, ΔP's$\to 0$). This has been provided by Bejan [3] as,

$$N_S = \ln\left[\left\{1-\varepsilon\left(1-\frac{T_2}{T_1}\right)\right\}\left\{1+\varepsilon\left(\frac{T_1}{T_2}-1\right)\right\}\right] \tag{31}$$

The remanent (flow-imbalance) irreversibility of two-stream parallel flow heat exchangers can be obtained by combining the equation of the entropy generation rate of the entire heat exchanger with the perfect design conditions and the effectiveness relation for parallel flow (Bejan [3]) as follows:

$$N_{S,imbalance} = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_2} = \ln\left[\left(\frac{T_2}{T_1}\right)^{C_r}\left\{1+\left(\frac{T_1}{T_2}-1\right)\frac{C_r}{1+C_r}\right\}^{1+C_r}\right] \tag{32}$$

For the extreme limit of imbalance ($C_r\to$infinite), the above equation becomes,

$$N_{S,imbalance} = \frac{T_2}{T_1} - 1 - \ln\left(\frac{T_2}{T_1}\right) \tag{33}$$

We can observe the above equation that the side 1 stream is so large that its temperature remains equal to T_1 from inlet to outlet. It behaves like a stream that condenses or evaporates at constant pressure. The remanent (flow-imbalance) irreversibility of two-stream counter flow heat exchangers can be obtained as follows

$$N_{S,imbalance} = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_2} = \ln\left[\left\{1-\frac{1}{C_r}\left(1-\frac{T_2}{T_1}\right)\right\}^{C_r}\frac{T_1}{T_2}\right] \tag{34}$$

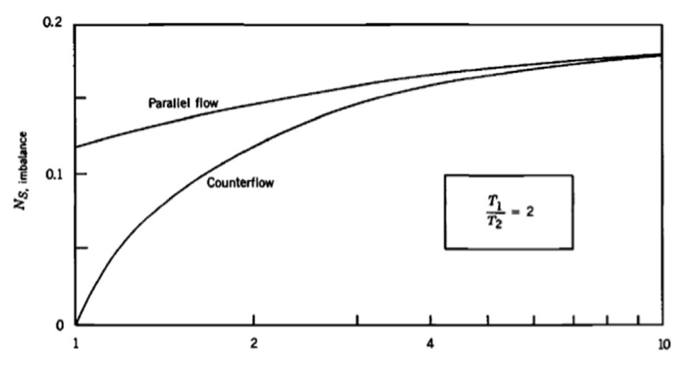

Fig. 4. The remanent irreversibility in parallel flow heat exchanger is greater than counter flow heat exchanger, where ω is the capacity rate ratio [2].

Observing the above equations, it is clear that the remanent (flow-imbalance) irreversibility in parallel flow is greater than in counter flow heat exchanger, as shown in Fig. 4.

The relative importance of the two irreversibility mechanisms is described by the irreversibility distribution ratio, which is defined by Bejan [15] as the ratio of fluid flow irreversibility to heat transfer irreversibility. This is defined with the help of Eq. (14) as,

$$\varphi = \frac{(\dot{S}_{gen})_{\Delta P}}{(\dot{S}_{gen})_{\Delta T}} \tag{35}$$

The total entropy generation rate equation becomes,

$$\dot{S}_{gen} = (1+\varphi)(\dot{S}_{gen})_{\Delta T} \tag{36}$$

The irreversibility distribution ratio increases as the irreversibility due to fluid friction contribution increases and decreases as irreversibility due to heat transfer increases. It provides the relative magnitudes of trade-off between the two important types

of irreversibilities. Because in heat exchangers as fluid flow increases, although heat transfer rate increases, at the same time pressure drop also increases. The optimum operation of heat exchanger can be obtained by the use of irreversibility distribution ratio expression. Bejan [16] proposed that counter flow heat exchangers, which very often serve as cycle components, can also serve a thermal insulation function. By promoting effective heat transfer in stream-to-stream direction, counter flow heat exchangers are effective insulators in the end-to-end direction. The paper proposes a conceptually new method for thermal insulation system optimization. The method, based on minimizing thermodynamic irreversibility, consists of externally controlling the variation of heat leak with temperature across the insulation.

The thermodynamic impact of an augmented technique can be evaluated by comparing it with unaugmented one. Bejan and Pfister [17] proposed that the merit of a given heat transfer augmentation technique may be evaluated by comparing the rate of entropy generation of the heat exchange apparatus before and after the implementation of the augmentation technique. They evaluated ducts with sand-grain roughness and transversal rib roughness. They defined the augmentation entropy generation number as the ratio of entropy generation rate after augmentation of flow passage to the entropy generation rate before the augmentation in the same flow passage as,

$$N_{s,a} = \frac{\dot{S}_{gen,a}}{\dot{S}_{gen,o}} \tag{37}$$

This technique was used to evaluate various heat transfer augmentation surfaces, including spiral tubes, twisted tape inserts, propeller inserts, and tubes with internal spiral ribs by Ouellette and Bejan [18]. The general form of augmentation entropy generation number is provided by Bejan [3] in terms of irreversibility distribution ratio, Φ as,

$$N_{S,a} = N_{S,\Delta T}\left(\frac{1}{1+\varphi}\right) + N_{S,\Delta P}\left(\frac{\varphi}{1+\varphi}\right) \tag{38}$$

The optimization of heat exchangers on the basis of entropy generation number was also carried out by Sarangi and Chowdury [19]. The maximum entropy generation rate paradox concept in non-dimensionalising the entropy generation rate equation with capacity rate point out using more appropriate means of non-dimensionalising. For nearly ideal heat exchanger with nearly balanced capacity rate, they obtained an expression for the number of entropy generation units, N_s. They compared the results of their expression with exact calculation and results of Bejan [10]. They observed that their new expression gave a much closer approximation and also could be easily incorporated into the new design procedure of Bejan. One of their results is provided in Fig. 5. The heat transfer irreversibility maximum is illustrated for an unbalanced counter flow heat exchanger.

A dimensionless entropy generation number obtained by dividing the entropy generation rate by the ratio of heat transfer to environment temperature seems to be the most rational method, and this approach was used by Witte and Shamsundar [20] as follows,

$$\eta_{W-s} = \frac{\dot{S}_{gen}}{(\dot{Q}/T_o)} \tag{39}$$

However, the introduction of a new parameter (the reference temperature) into the analysis leads to further complications along with other temperature terms namely inlet and outlet temperature of hot and cold fluids. Optimization of heat exchangers based on entropy generation is carried out by Sekulic and Herman [21]. They applied this concept in the core sizing procedure of a compact cross flow heat exchanger for gas-to-gas application. In the final analysis, the approach objective was the pressure drop choice in such a way that from the total set of possible heat exchanger core dimensions the thermodynamically optimal one was selected. A sample of their work is provided in Fig. 6. The overall Ntu constraint is listed on each of the 'V' shaped curves.

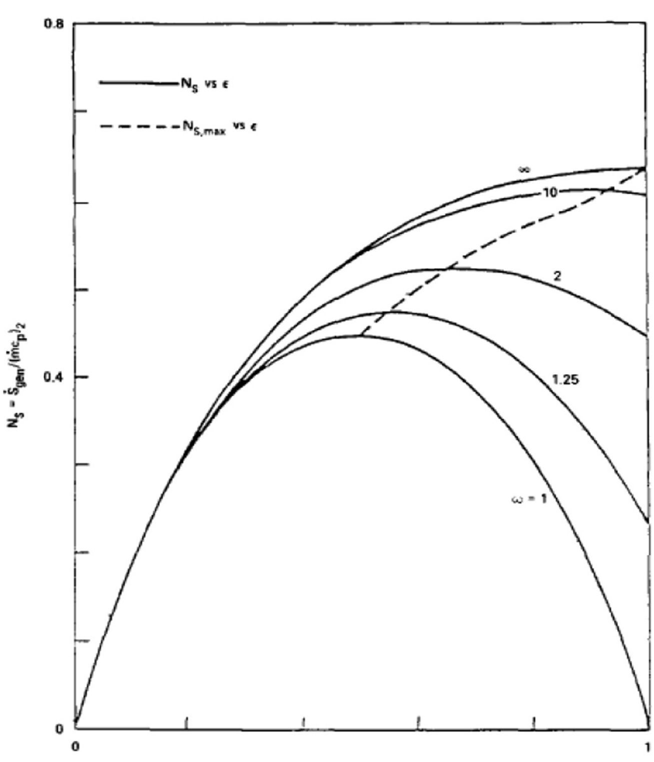

Fig. 5. The irreversibility of an imbalanced counter flow heat exchanger with negligible pressure drop irreversibility, where ω is the capacity rate ratio and T_1/T_2=0.25 [19].

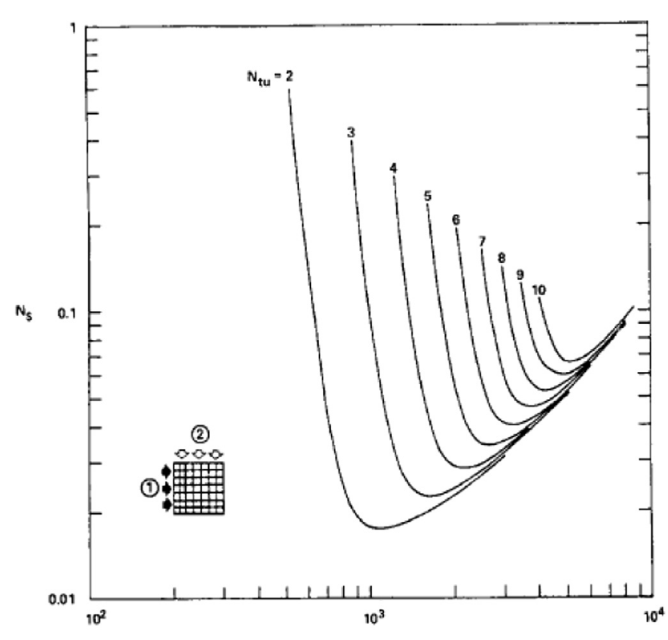

Fig. 6. Minimization of entropy generation subject to constant Ntu for a cross flow heat exchanger [21].

As Ntu increases, the optimum slenderness ratio of one side of the surface increases.

Optimization of heat exchangers based on entropy generation is carried out for counterflow and crossflow heat exchangers by Sekulic [22]. In their analysis, the quantity termed enthalpy exchange irreversibility norm (*EEIN*) was the measure of the internal heat exchanger irreversibilities. The researcher discussed the behavior of *EEIN* as a function of the heat exchanger thermal size for an arbitrary flow arrangement and more precisely for two characteristic limiting cases: cocurrent and countercurrent heat exchangers.

Optimization of heat exchangers based on entropy generation is carried out by Grazzini and Gori [23] for counter flow heat exchangers. Their expression was applicable to incompressible liquids and perfect gases. They defined two new entropy generation numbers, N_Q and N_M as follows,

$$N_Q = \frac{\dot{S}_{gen}}{\Delta S_Q} \tag{40}$$

where ΔS_Q is interpreted as the entropy generated if the heat transferred in the heat exchanger is exchanged between the inlet temperatures of the two streams.

$$N_M = \frac{\dot{S}_{gen}}{\Delta S_{max}} \tag{41}$$

where ΔS_{max} is the maximum entropy generation is due to the free expansion of the fluid from the inlet pressure only to a very low reference pressure. They investigated the relative position of both the maximum and minimum in the entropy generation numbers. They applied their analysis to an air-air counter-current heat exchanger. The three entropy generation numbers, N_S, N_Q and N_M, had a different variation with Ntu at the different values of the capacity flow rate ratio employed in the calculations.

Analysis of heat exchangers can be carried out by Bejan number with the use of entropy generation rate due to heat transfer and pressure drop. Bejan number is defined as the ratio of entropy generation due to heat transfer to total entropy generation as given by Paoletti et al. [24].

$$Be = \frac{(\dot{S}_{gen})_{\Delta T}}{(\dot{S}_{gen})_{\Delta T} + (\dot{S}_{gen})_{\Delta P}} = \frac{1}{1+\varphi} \tag{42}$$

Bejan number ranges from 0 to 1. Accordingly, $Be = 1$ is the limit at which the heat transfer irreversibility dominates, $Be = 0$ is the opposite limit at which the irreversibility is dominated by fluid friction effects, and $Be = 1/2$ is the case in which the heat transfer and fluid friction entropy generation rates are equal.

Sekulic [25] suggested the use of quality level of energy transformation to evaluate the performance of heat exchangers, and called this concept as Quality of Energy Transformation. According to this concept, entropy generation equal to zero (reversible process) corresponds to the highest quality, and the quality of energy transformation decreases with increase of entropy generation. They used the entropy generation caused by finite temperature differences, scaled by the maximum possible entropy generation that could exist in an open system with two fluids, as the quantitative measure of the quality of energy transformation (the heat exchange process). This quality was defined as

$$\text{Quality of energy transformation} = \frac{1 - (\text{Entropy generation in the real process})}{(\text{Entropy generation in the most disadvantageous case})} \tag{43}$$

Also, point that should be considered was that the use of this concept required the determination of the most disadvantageous case. Substituting the entropy generation number and the maximum possible dimensionless entropy generation into above equation gives the following quantity termed as Heat Exchange Reversibility Norm (*HERN*):

$$Y_S = 1 - \frac{N_S}{N_{S,\,max}} \tag{44}$$

Performance evaluation criteria equations based on the first law analysis and entropy production concept have been developed by Zimparov and Vulchanov [26]. Extended performance criteria based on the augmentation entropy generation numbers for enhanced heat transfer surfaces for ducts with constant wall temperature was developed by Zimparov [27]. Also, extended performance criteria based on the augmentation entropy generation numbers for enhanced heat transfer surfaces for ducts with constant heat flux was developed by Zimparov [28]. These criteria were applied to 10 spirally corrugated tubes in order to determine the benefits of these enhanced tubes as augmentation techniques, and optimal operational and design conditions were determined. The meaning of extended performance criteria is that, the purpose of the work in [27,28] is to extend the performance evaluation criteria equations discussed previously in [26] including the effect of fluid temperature variation along the length of a tubular heat exchanger and to add new information to entropy generation minimization method, assessing two objectives simultaneously namely entropy generation due to heat transfer and fluid friction. Heat transfer enhancement devices increase the rate of heat transfer, but they also increase the friction factor associated with the flow. This raises the question of how to employ enhancement techniques in order to minimize the overall entropy generation associated with the heat exchanger operation.

A study on the optimization of the convective heat transfer in a packed duct with constant wall temperature was conducted by Demirel [29]. The entropy generation number was used as a criterion for system optimization, and it was found that this number might be minimized through a proper selection of operating conditions and design parameters of the system. Bejan and Errera [30] gave the solution to the fundamental problem of how to maximize the mechanical power extracted from a hot single-phase stream when the total heat transfer area bathed by the stream is constrained. They showed that the optimization has two degrees of freedom: the shape of the stream temperature distribution as a function of the length traveled along the heat transfer surface and the position of this distribution on the absolute temperature scale. The effect of the sizes and capacity rates of the two heat exchangers is analyzed.

Ogulata et al. [31] carried out irreversibility analysis of cross flow heat exchangers. They tested this new heat exchanger with an applicable experimental set up, considering temperatures, velocity of the air and the pressure losses occurring in the system. They measured these variables and determined the efficiency of the system. They took into consideration the irreversibility of the heat exchanger while they performed the heat exchanger design so that the minimum entropy generation number had analyzed with respect to second law of thermodynamics in the cross-flow heat exchanger. A comparison is presented of the main approaches to second law analysis by Rosen [32]. They stated that exergy often is a consistent measure of economic value (i.e., a large quantity of exergy is often associated with a valuable commodity) while energy is only sometimes a consistent measure of economic value. In addition, several significant implications of second law analysis are examined in the fields of environmental impact and economics.

Ordonez and Bejan [33] showed that the main architectural features of a counter flow heat exchanger can be determined based on thermodynamic optimization subject to volume constraint. It is assumed that the channels are formed by parallel plates, the two fluids are ideal gases, and the flow is fully

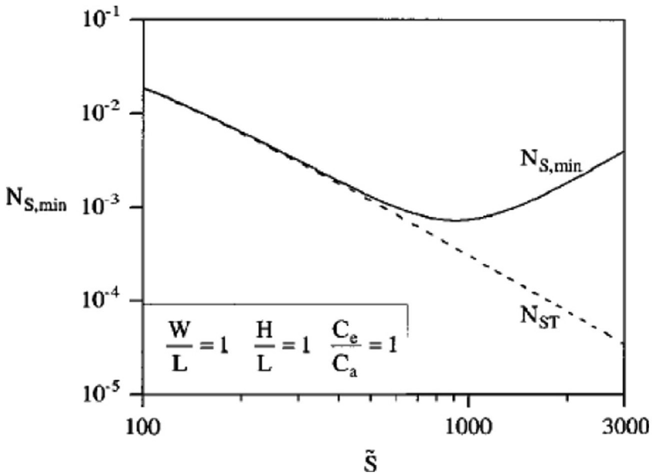

Fig. 7. The effect of total contact area $\tilde{S}$ on the entropy generation rate corresponding to the geometry optimized. Where L, H and W are the dimensions of plate heat exchanger and C is the capacity rate of cold (a) and hot (e) streams [33].

developed, laminar or turbulent. They showed that the irreversibility of the heat exchanger core was minimized and the design could be optimized with respect to the ratio of the two-channel spacings, the total heat transfer area between the two streams and the ratio of the capacity rates of the two streams as shown in Fig. 7 of their sample work and charts for calculating the corresponding dimensions are reported in [33]. The optimized features of the geometry were robust with respect to whether the external discharge irreversibility was included in the entropy generation rate calculation. Hesselgreaves [34] suggested dividing the entropy generation rate by the ratio of heat transfer to inlet temperature of cold stream to eliminate the disadvantages of the method described earlier [20]. The advantage of this method is that entropy generation number can exceed unity.

$$N_s = \frac{\dot{S}_{gen}}{(\dot{Q}/T_{c,in})} \tag{45}$$

The types of heat exchangers analyzed were: heat exchangers with flow imbalance, balanced counter flow, parallel flow, condensing on one side, and evaporation on one side. An important result of this investigation is that the basic entropy generation relationship for gas flows is controlled by the flow Mach number.

Enhancement of heat transfer by a combination of three-start spirally corrugated tubes with a twisted tape was analyzed by Zimparov [35]. These criteria were applied to 10 spirally corrugated tubes in order to determine the benefits of these enhanced tubes as augmentation techniques, and optimal operational and design conditions were determined. Equipartition of forces as a lower bound on the entropy production in heat exchange concept is used by Nummedal and Kjelstrup [36]. Theoretical analysis and experimental confirmation for the principle to improve the thermal performance of heat exchangers is performed by Guo et al. [37]. A concept of temperature difference field (*TDF*) has been proposed to study the effect of the temperature difference distribution between the hot and cold fluids on the performance of heat exchangers. For convenience, in discussing the effects of the uniformity of *TDF* on effectiveness, a parameter describing the degree of uniformity of *TDF* is defined as the uniformity factor of *TDF*. But, Guo et al. [37] concept is a copy (i.e. not original) version of Bejan's [16] principle of how to distribute the ΔT in a heat exchanger. Bejan's [16] version is $\Delta T/T$=constant, where T is the absolute temperature (and, because in most applications $T \gg \Delta T$, then the $\Delta T/T$=constant becomes ΔT=constant). In other words, Guo et al. [37] published as “new” the $T \gg \Delta T$ limit of Bejan's [16] “general” principle, which was first published in 1979. Also, Bejan's original version appears on pages 180–182 of his 1982 book [1], and on pages 161–163 of his 1996 book [2]. It also appears in all three editions of Bejan's textbook Advanced Engineering Thermodynamics [3–5].

Further, clarification regarding lack of additional information with respect to thermodynamics is provided by Bejan [38]. It is illustrated in Fig. 4 in the reference [37]. Shown is that the heat exchanger with uniform ΔT (Fig. 4(b)) is better than the heat exchanger with nonuniform ΔT (Fig. 4(a)). This aspect is not new, it has been published in 1979 by Bejan based on variational calculus presented in reference Bejan [16] as discussed above. The version (ΔT=constant) presented by [37] is nothing more than the special case of result presented by Bejan [16] in the limit $\Delta T \ll T$, which is common in heat exchanger design, but not in cryogenics.

Grazzini et al. [39] thermodynamically analyzed entransy with respect to entropy and concluded that firstly, the entransy have got some difficulties in its definition. It has actually been introduced in the literature following different approaches. The first was based on a cycle and the other one on heat exchange. Both these approaches have physical difficulties, and the results obtained with the entransy approach are also obtainable by the well-known entropy approach.

Herwig [40] carried out the critical assessment of newly introduced quantity entransy, basically on the background of the classical concept of heat transfer and with respect to the two questions of whether the new quantity is consistent with the generally accepted concept of heat transfer and whether there is a need for the new quantity. The investigation concluded that the both the prerequisites from the critical analysis are not fulfilled to an extent that would justify introducing entrancy as a general new quantity in the heat transfer analysis.

The duality of heat exchangers to act as both heat conductors and insulators when considering heat exchanger performance is studied on the basis of entropy generation by Ogiso [41]. By defining an appropriate index of the entropy generation rate, the performance paradox pointed out by Bejan can be resolved. The case of a balanced counter-flow system was examined and it has been shown that a heat exchanger can be regarded as both a heat conductor and a heat insulator in terms of entropy generation. In order to define the entropy generation rate in terms of heat conducting capacity, a quantity which is non-dimensional is introduced as,

$$\Gamma = \dot{S}_{gen}/AU = (\dot{S}_{gen}/C_{min})/(AU/C_{min}) = N_S/Ntu \tag{46}$$

In the limit of $\varepsilon \to 0$ (no heat exchange), the heat exchanger would become a perfect heat insulator. As Ntu (0 to ∞) is a direct measure of heat conductor performance, $1/Ntu$ (∞ to 0) represents a measure of heat insulator performance. The factor is Ntu, as is easily understood. $Ntu \to 0$ indicates a perfect insulator and $Ntu \to \infty$ represents the worst heat insulator. These results have been shown in Fig. 8, which is considered as an example to explain Eq. (46) as provided in [41].

The analytical solutions for the temperature variation of two streams in parallel flow, counter flow and cross-flow heat exchangers and related entropy generation due to heat exchange between the streams were analyzed by Galovic et al. [42]. The analysis of limiting cases for the relative entropy generation is performed, and the corresponding analytical expressions are given. The relative entropy generation is defined as a ratio of actual entropy generation rate and its maximal value.

$$N_s = \frac{\dot{S}_{gen}}{(\dot{S}_{gen})_{max}} \tag{47}$$

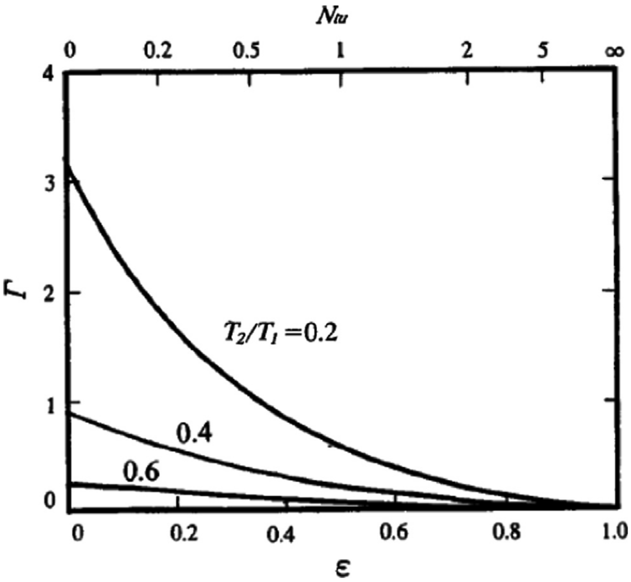

Fig. 8. Behavior of non-dimensional quantity of Eq. (46) versus effectiveness and *Ntu* [41].

Further, analysis of obtained expressions is carried out for the case of evaporators or condensers. The entropy generation is related to the heat exchanger effectiveness, so the heat exchangers can be evaluated quantitatively. Obtained results can be used in a more general procedure for heat exchanger optimization.

Shah and Skiepko [43] have shown that the heat exchanger effectiveness can be maximum, having an intermediate value or minimum at the maximum irreversibility operating point depending on the flow arrangement of the two fluids. Similarly, the heat exchanger effectiveness can be minimum or maximum at the minimum irreversibility operating point. They have discussed the heat exchanger performance and irreversibility trends by combining the temperature difference irreversibility with the *Ntu* results for complex flow arrangements. They concluded that the irreversibility curves for classical flow arrangements may not be applicable to study irreversibility behavior for complex flow arrangements. Regarding the overall exchanger effectiveness for complex flow arrangement under study, the corresponding trends by component subexchanger performances are provided. It appears from the analysis that pure counter flow subexchangers when operating in a complex flow arrangement can have significantly worse performance while pure parallel flow ones can improve the exchanger performance of complex flow arrangements. It should be emphasized that in principle one may decompose readily a given complex flow arrangement into the elemental units. However, irreversibility analysis of complex flow arrangements cannot be done using the results obtained for subsequent elemental subexchangers when operating standalone. The only correct way to get readily the final results is to follow the approach presented in their paper that disregarding standalone behavior takes into account individual subexchanger flow arrangements and how the subexchangers are connected and what are consequences of the coupling for irreversibility and effectiveness behavior.

The entropy generation minimization method used to optimize a single-phase, convective, fully developed flow with uniform and constant heat flux was carried out by Ratts and Raut [44]. For fixed mass flow rate and fixed total heat transfer rate, and the assumption of uniform and constant heat flux, an optimal Reynolds number for laminar and turbulent flow is obtained. The study also compares optimal Reynolds number and minimum entropy generation for cross sections: square, equilateral triangle, and rectangle with aspect ratios of two and eight. The rectangle with aspect ratio of eight had the smallest optimal Reynolds number, the smallest entropy generation number, and the smallest flow length.

In another paper, Shah and Skiepko [45] has explained the interrelationship between the temperature difference irreversibility and heat exchanger effectiveness to clarify the performance trends of exchangers with some complex flow arrangements. They concluded that the disadvantage of complex arrangements is reverse heat transfer that may occur within such exchangers. It is called as internal temperature cross and beyond that point along the flow length $T_{c,out} > T_{h,out}$, and reverse heat transfer takes place (original cold fluid transferring heat to the original hot fluid). Based on the 1st law analysis, one can find the starting point of reverse heat transfer by the solution of differential energy transfer equations in terms of temperature distributions of both the fluids. Hence, finding the internal temperature cross for a heat exchanger with complex flow arrangement under consideration is not an easy task. However, if the irreversibility analysis is applied, the temperature cross phenomena can be easily detected due to the peculiar behavior of irreversibility versus *Ntu* curve going through minimum in between two maximum peaks due to reverse heat transfer in the exchanger.

In the design of a heat exchanger, different loss mechanisms like pressure drop and parasitic heat flows are often treated separately. Acceptable values for the pressure drop and total heat leakage were estimated by Lerou et al. [46] and thus counter flow heat exchanger geometry is more or less arbitrarily chosen. They studied optimization of counter flow heat exchanger geometry through minimization of entropy generation. In their study, the investigators applied another, less familiar design strategy where different loss mechanisms such as pressure drop and parasitic heat flows were all treated as a production of entropy. Thus, it was possible to compare and sum them. In this way, they found that a heat exchanger configuration was optimal for a certain application, producing a minimum of entropy and thus had minimum losses.

Entropy minimization principle can dictate uniform local entropy generations along the heat exchanger in order to minimize the total entropy generation rate due to only heat transfer as discussed by Balkan [47]. According to this principle, the local entropy generation rate should be kept constant along the heat exchanger to minimize the total entropy generation rate. For a certain heat duty and area of an existing exchanger, this is done by changing the temperatures of one fluid while the temperatures of the other fluid are held constant. Since the heat duty is fixed, the change in the temperatures of the fluid after the change, however, may sometimes cause a drastic change in its flow rate. This may cause considerable changes in the overall heat transfer coefficient and, consequently, in the entropy generation rate. The principle of this concept with variable overall heat transfer coefficient is applied to some cases of heat exchange, and a simple method is presented as a criterion for the proper choice of the fluid to be changed.

A combination of the first and second laws of thermodynamics has been utilized in analyzing the performance of a double pipe heat exchanger with a porous medium attached over the inner pipe by Allouache and Chikh [48]. The numerical procedure is based on the control volume method and allowed to obtain the velocity and temperature fields and thus to compute the rate of entropy generation. The effects of the porous layer thickness, its permeability, the inlet temperature difference between the cold and hot fluids, and the thermal conductivity ratio are documented. They used Bejan Number expression in the analysis.

The theoretical and experimental results of the second law analysis on the heat transfer and fluid flow of a horizontal concentric tube heat exchanger are presented by Naphon [49].

They analyzed the influences of the inlet conditions of both working fluids flowing through the heat exchanger on the heat transfer characteristics, entropy generation, and exergy loss. Based on the conservation equations of energy, they developed the mathematical model and solved using the central finite difference method to obtain temperature distribution, entropy generation, and exergy loss. The predicted results obtained from the model were validated by comparing with experimental measured data. From this comparison, they found that there was reasonable agreement between predicted results and those from the measured data.

Analysis of heat transfer and fluid flow thermodynamic irreversibilities is realized on an example of a counter flow double pipe heat exchanger utilizing turbulent air flow as a working fluid was carried out by Mohamed [50]. During the process of mathematical model creation and for various working and constructing limitations, the researcher studied total thermodynamic irreversibility. Their work proved that the irreversibility occurred due to unequal capacity flow rates (flow imbalance irreversibility). They concluded that the heat exchanger should be operated at effectiveness, $\varepsilon > 0.5$ and the well operating conditions would be achieved when $\varepsilon \sim 1$ where low irreversibility was expected. They adopted a new equation to express the entropy generation numbers for imbalanced heat exchangers of similar design with smallest deviation from the exact value. The results obtained from the new equation are compared with the exact values and with those obtained by Bejan.

The use of entropy generation minimization allows the combined effect of heat transfer and pressure drop to be assessed through the simultaneous interaction with the tube bank. A general dimensionless expression for the entropy generation rate is obtained by Khan et al. [51], considering a control volume around a tube bank and applying the conservation equations for mass and energy with the entropy balance. Considering a control volume around a tube bank and applying the conservation equations for mass and energy with the entropy balance they defined N_S as,

$$N_S = \dot{S}_{gen}/(Q^2 V_{max}/k_f \nu T_o^2) \tag{48}$$

where V_{max} is maximum velocity in minimum flow area, k_f is the thermal conductivity of fluid, ν is the kinematic viscosity of the fluid. In their study, both inline and staggered arrangements were studied and their relative performance was compared for the same thermal and hydraulic conditions. They employed the cross flow correlations for the heat transfer and pressure drop to calculate entropy generation rate. They obtained a general dimensionless expression for the entropy generation rate by considering a control volume around a tube bank and applying conservation equations for mass and energy with entropy balance. Empirical correlations for heat transfer coefficients and friction factors were used, where the characteristic length was used as the diameter of the tubes and reference velocity used in Reynolds number and pressure drop was based on the minimum free area available for the fluid flow. They showed that all relevant design parameters for tube banks, including geometric parameters and flow conditions, could be simultaneously optimized.

Kurtbas et al. [52] studied the effects of propeller-type turbulators located in the inner pipe of co-axial heat exchanger investigated on entropy generation rate and exergy loss rate. The experiments were performed with different distances of turbulators. In this system, they investigated heat transfer, entropy generation rate, and exergy loss rate. Then, they investigated the influences of angle, diameter, and number of the blades on the heat transfer, entropy generation rate, and exergy loss rate and compared with each other for various values of the Reynolds number. They found that Nusselt number and exergy loss rate approximately increased. Also, heat exchanger efficiency increased. Entropy generation number is obtained by the ratio of entropy generation rate of rough tube to the entropy generation rate of smooth tube. They also defined exergy loss rate as follows:

$$E^* = \frac{\dot{I}}{\dot{Q}} \tag{49}$$

Relations between entropy generation number, exergy loss rate and effectiveness with other parameters were found.

The effect of flow geometry parameters on transient entropy generation for turbulent flow in a circular tube with baffle inserts has been investigated by Tandiroglu [53]. Different flow geometry parameters of pitch to diameter ratio, baffle orientation angle, ratio of smooth to baffled cross-section area and ratio of tube length to baffle spacing were varied parametrically during the experiments. One smooth tube and nine different baffle inserted tubes geometries were tested. The time averaged entropy generation corresponding to the flow geometry parameters was compared under the condition of constant heat flux. The general empirical correlation of the time averaged entropy generation was developed and considered to be applicable within the range of Reynolds number $3000 < Re < 20{,}000$.

Second law analysis of a cross-flow heat exchanger is studied in the presence of a balance between the entropy generation due to heat transfer and fluid friction by Kotcioglu et al. [54]. It was found that increasing the cross-flow fluid velocity enhances the heat transfer rate and reduces the heat transfer irreversibility. A new equation is adopted to express the entropy generation numbers for imbalanced heat exchangers of similar design with the smallest deviation from the exact value. The results obtained from the new equation are compared with the exact values and with those obtained by Bejan. They investigated the entropy generation in a cross-flow heat exchanger with a new winglet-type convergent–divergent longitudinal vortex generator. They presented optimization of heat exchanger channel geometry and effect of design parameters regarding the overall system performance. They found that increasing the cross-flow fluid velocity enhanced the heat transfer rate and reduced the heat transfer irreversibility.

Experimental and theoretical investigations on the entropy generation of a horizontal concentric micro-fin tube heat exchanger are presented by Naphon [55]. Effect of relevant parameters on the entropy generation number is considered. The results also showed that the relevant parameters have significant effect on the entropy generation, entropy generation number, and exergy loss. The effects of the inlet conditions of both working fluids flowing through the heat exchanger on the heat transfer characteristics and entropy generation are also discussed. The experiments setup are designed and constructed for the measured data by using hot water and cold water as working fluids. The micro-fin tube is fabricated from the copper tube with an inner diameter of 8.92 mm. The experiments are performed for the hot and cold water mass flow rates in the range of 0.02–0.10 kg/s. The inlet hot water and inlet cold water temperatures are between 40 and 50 °C, and between 15 and 20 °C, respectively. The effects of relevant parameters on the entropy generation and exergy loss are discussed. A central finite difference method is employed to solve the model for obtaining temperature distribution, entropy generation, and exergy loss of the micro-fin tube heat exchanger. The predicted results obtained from the model are verified by comparing with the measured data. Reasonable agreement is obtained from the comparison between predicted results and those from the measured data.

Cheng et al. [56] analyzed heat exchangers based on the concepts of the entransy dissipation, the entropy generation and the entransy–dissipation-based thermal resistance of multi-stream heat exchangers. The physical meaning of the principle of

the uniformity of temperature difference field is discussed for the two-stream heat exchangers, and the uniformity principle of temperature difference field is proved.

Detailed differences between the entropy generation and entransy dissipation are provided in Bejan [38]. It can be found as Problem 1.1 in Bejan [1], or as the solved Example 2.1 in Bejan [5]. Two identical bodies (mass m, specific heat c) are initially at different temperatures, $T_e+\Delta T/2$ and $T_e-\Delta T/2$. The initial temperature difference between them is ΔT, and over this finite (narrow) temperature interval c can be assumed constant. The two bodies, together, form an isolated system. The bodies are placed in thermal contact, and after a sufficiently long time they reach thermal equilibrium. For this process, the laws of thermodynamics make two statements.

Applying the first law of thermodynamics which defines the conservation of energy as,

$$U-U_o=\dot{m}c(T-T_o) \tag{50}$$

$$\dot{m}c\left(T_e+\frac{\Delta T}{2}-T_o\right)+\dot{m}c\left(T_e-\frac{\Delta T}{2}-T_o\right)=2\dot{m}c(T_f-T_o) \tag{51}$$

where T_f is the final temperature of the two bodies. The first law requires that $T_f=T_e$. The second law defines the irreversibility of the process, or the entropy generation as,

$$\dot{S}_{gen}=2\dot{m}c\ \ln\left(\frac{T_e}{T_o}\right)-\dot{m}c\ \ln\left(\frac{T_e+\Delta T/2}{T_o}\right)-\dot{m}c\ \ln\left(\frac{T_e-\Delta T/2}{T_o}\right)\geq 0 \tag{52}$$

For analytical simplicity it is assumed $\Delta T \lll T_e$, and the entropy generation Eq. (52) becomes,

$$S_{gen}=\dot{m}c\left(\frac{\Delta T}{2T_e}\right)^2 \tag{53}$$

This quantity has physical significance because it is proportional to the work that could have been produced by a reversible engine operating between the two bodies throughout the process. In the limit $\Delta T \lll T_e$, that the lost work is [1,5],

$$W_{lost}=\frac{1}{4}\dot{m}c\frac{(\Delta T)^2}{T_e} \tag{54}$$

The quantity entransy provided by [57], is defined as a isothermal thermodynamic system (a solid body) that is closed and capable of experiencing only heat interactions with its environment. The referenced system of [57] has the temperature T, mass M, specific heat at constant volume C_v, and defined the quantity as,

$$Q_{vh}=MC_vT \tag{55}$$

This is called as thermal energy of the heat stored in an object with constant volume, which may be referred to as thermal change. Reference [57] defined a heating process in which the system experiences an increase in temperature, T and correspondingly, in Q_{vh} of above Eq. (55). Then, these two effects are arbitrarily multiplied in order to define a quantity called potential energy that does not have the units of energy,

$$E_{vh}=\int_0^T Q_{vh}dT=\int_0^T MC_vTdT=\frac{1}{2}MC_vT^2 \tag{56}$$

Now, the above discussed example of Bejan [1,5] is applied to entransy concept, which according to second law of thermodynamics defined that during the process the two-body system losses entransy as,

$$E_{vh,lost}=E_{vh,before}-E_{vh,after}$$

$$E_{vh,lost}=\frac{1}{2}mc\left(T_e+\frac{\Delta T}{2}\right)^2+\frac{1}{2}mc\left(T_e-\frac{\Delta T}{2}\right)^2-\frac{1}{2}2mcT_e^2$$

$$E_{vh,lost}=\frac{1}{4}mc(\Delta T)^2 \tag{57}$$

Referring to Eqs. (54) and (57) show that the entransy dissipation is simply a multiple of the dissipated available work, or the entropy generation of Eq. (53).

Cheng et al. [56] discussions on entransy is based on concept introduced by Guo et al [37] and Guo et al [57] in order to publish as new concept of all the original advanced that have been published before them with the methods of entropy generation minimization, exergy analysis and the constructal law [1–6]. In brief, "entransy" is the completely arbitrary decision (with zero basis in physics) to regard T^2 as a new thermodynamic property (called "entransy") that represents "the ability of a solid body to have heat transfer". Note: T^2, in place of T. It should be obvious that with such originality it is immediately possible to replace with "entransy dissipation minimization" all the existing results of entropy generation minimization, exergy dissipation minimization, and resistance minimization. Also, here to mention that [37,56,57] could have defined just as arbitrarily the entransy is T^n, where $n>0$, and could have published even more "entransy" rip offs of existing publications, one "new" entransy paper for every n.

Further, clarification regarding entropy versus entransy is provided in Grazzini et al. [39]. The entransy function is analyzed critically in relation to the classical thermodynamic approach in order to understand its physical fundamentals. They concluded that the entransy analysis does not contain any new information in comparison with a classical thermodynamic analysis of systems. Furthermore, they showed that entransy dissipation does not convey any more information than entropy generation. Grazzini et al. [39] discussed that in [57] an analogy is made with electrical phenomena and an equation written for entransy dissipation in a thermal resistor in analogy with electric dissipation. But the electricity is converted into thermal energy, downgraded from work to heat, while what entransy becomes is difficult to understand. Indeed, a formal analogy is not enough to obtain a physical one. In sum, any result obtained by entransy dissipation analysis is simply a duplicate of the entropy generation approach, which is well known and has been in textbooks since 1982 [1]. The thermodynamic approach to irreversibility proved the entropy generation extremum theorem [1], which is the basis of Bejan's constructal theory [4].

Leong et al. [58] investigated heat transfer and entropy analysis of three different types of heat exchangers operated with nanofluids. The influence of arrangement of baffles on these heat exchangers was studied. The study focused on the heat transfer and entropy generation comparison of segmental and helical type shell and tube heat exchangers. Their study focused on the heat transfer and entropy analysis of segmental, 25° and 50 helical baffles shell and tube heat exchangers. Furthermore, the entropy generation is estimated to evaluate the suitability of nanofluid as a coolant in a heat exchanger. Study also focused on comparative heat transfer and entropy generation performance of segmental and helical shell and tube heat exchangers operated with nanofluids. The results from their study concluded that 25° helical baffles shell and tube heat exchanger exhibits the highest overall heat transfer coefficient with the increase of mass flow rate followed by segmental and 50° helical. Apart from that, overall heat transfer coefficient augments with the addition of nanoparticles irrespective of the type of heat exchanger. Heat transfer rate of the 25° helical baffles heat exchanger is the highest compared to that of segmental and 50° helical exchanger. Entropy generation is minimized for the 50° helical exchanger due to its lowest total pressure drop.

3.2. Exergy analysis as performance parameter

Rational efficiency (second law effectiveness) of heat exchangers was introduced by Bruges [59] and is defined as the ratio of

availability (exergy) gained by the cold stream to availability (exergy) loss of the warm stream. Rational efficiency in general is defined as the ratio of exergy output to exergy input. It is a dimensionless measure of heat exchanger irreversibility and is given as,

$$\psi = \frac{\sum \Delta \dot{E}_{out}}{\sum \Delta \dot{E}_{in}} \tag{58}$$

The rational efficiency is more practical, as it takes care of all the irreversibilities occurring in the system in a single expression. It is also known as exergetic efficiency. Rational efficiency is a criterion of performance which can be formulated for a plant or a plant component. The ratio of exergy output to exergy input is less than one, the difference depending on the degree of irreversibility of the process; for full reversibility, the ratio is one. This feature of the ratio makes it particularly suitable as a criterion of degree of thermodynamic perfection of a process [11]. In formulating the traditional criteria of performance like thermal efficiency and coefficient of performance, all forms of energy are taken as equivalent, in which no reference is made to second law of thermodynamics.

Golem and Brzustowski [60] examined the irreversibility of heat exchangers using the rational effectiveness, and extended this concept to the local level. The meaning of local level is component wise (as an example heat exchanger) analysis of bigger assembly of a system. Exergetic efficiency is considered as a general criterion of performance that can be formulated for any steady state process taking place in a system or its component, which produces a useful output expressible in terms of exergy. Mukherjee et al. [61] proposed the use of Merit Function to evaluate heat exchangers, and defined the merit function as the ratio of exergy transferred to the sum of exergy transferred and exergy destroyed.

$$M = \frac{\dot{E}_a}{\dot{E}_a + \dot{I}} \tag{59}$$

where

$$\dot{E}_a = \dot{Q}\left(1 - \frac{T_o}{T_w}\right) \tag{60}$$

is the rate of exergy transfer accompanying energy transfer at the heat transfer rate. T_o is the reference temperature, which is considered as exergy reference environment temperature, and T_w is the wall temperature which is considered as a suitable temperature at the surface where the heat transfer takes place. Increase in the merit function means that the rate of increase in heat transfer well exceeds the rate of increase in irreversibility; therefore, higher values of the merit function are preferred. The second law analysis of heat transfer in swirling flow through a cylindrical duct was carried out. The influence of the swirling flow on the merit function was discussed. The study emphasized that the merit function analysis could be extended to other convective heat transfer configurations associated with different types of internal turbulence promoters for selection of the best-fitted insert for a particular process.

A different form of dimensionless irreversibility parameter is non-dimensional exergy destruction proposed by Prasad and Shen [62] as,

$$\Delta\psi = \frac{\dot{I}}{\dot{m} c_p T_o} \tag{61}$$

The non-dimensional exergy destruction in a tubular heat exchanger in which the tube wall is assumed to be at a constant temperature was used for the analysis. By minimizing the non-dimensional exergy destruction, a thermodynamically optimum can be determined for the operation of a given heat exchanger, and the augmented surface with minimum non-dimensional exergy destruction can be selected as the most preferable configuration. Fig. 9 shows the results of dimensionless exergy destruction as a function of Reynolds number for coiled-wire inserts compared to smooth tube, where CPI is coils per inch.

It can be accessed that the use of an augmentation device results in an improved heat transfer coefficient, thus reducing exergy destruction due to heat transfer. But there will be an additional resistance to fluid flow resulting in an increase in exergy destruction due to frictional effects. Therefore, the exergy destruction number, which is the ratio of the non-dimensional exergy destruction number of the augmented surfaces to that of the unaugmented one, can be used to evaluate heat transfer enhancement devices as provided by Prasad and Shen [63].

$$N_E = \frac{I_a^*}{I_s^*} \tag{62}$$

where the subscript 'a' term corresponds to non-dimensional exergy destruction for augmented surface and subscript 's' term corresponds to non-dimensional exergy destruction for smooth surface. By using the exergy destruction number for the augmented surface performance analysis, we must be able to achieve that non-dimensional exergy destruction must be minimum and exergy destruction number must be less than or equal to unity. The augmented system would be thermodynamically advantageous only if the exergy destruction number is less than unity, because the values of exergy destruction number less than unity means that the exergy destruction in the augmented case becomes less than the unaugmented one. The exergy analysis method including the non-dimensional exergy destruction, the exergy destruction number, and the heat transfer improvement number were used to determine the performance of several wire-coil inserts in forced convection heat transfer.

Specific irreversibility is defined as the ratio of the irreversibility to the thermal exergy rate of fluid at the inlet of a heat exchanger as provided by Das and Roetzel [64].

$$I_{sp} = \frac{\dot{I}}{\dot{E}_{in}} \tag{63}$$

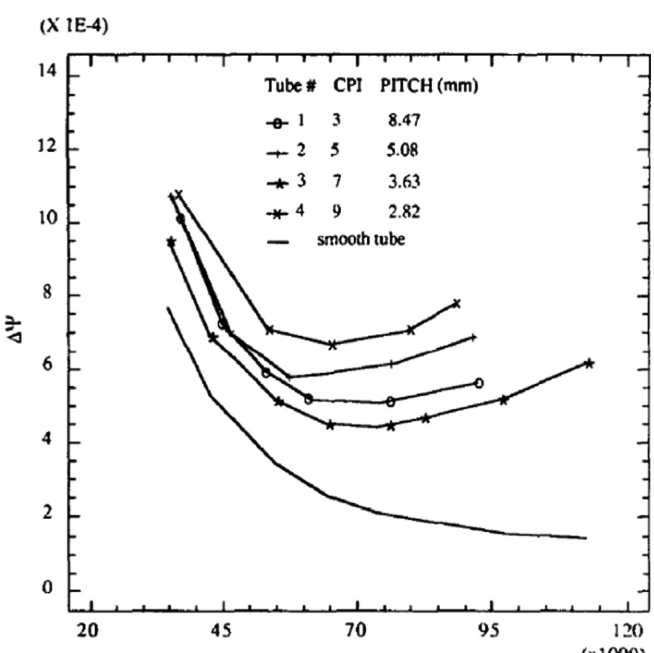

Fig. 9. Dimensionless exergy destruction ($\Delta\psi$) as a function of Reynolds number for coiled-wire inserts compared to smooth tube, where *CPI* is coils per inch [62].

In the above expression the denominator represents the thermal exergy rate of fluid at the inlet of a heat exchanger. The highest performance for heat exchangers can be obtained by minimizing the specific irreversibility. They presented a second law analysis for thermally dispersive flow through a plate heat exchanger using the specific irreversibility. A second law optimization was conducted for heat transfer equipments as presented by using a single-pass plate heat exchanger. Fig. 10 shows the variations of different specific irreversibilities which includes due to heat transfer and pressure and effectiveness with respect to *Ntu*.

A study on second law analysis of a swirling flow in a circular duct with restriction was investigated by Yilbas et al. [65]. The governing fluid and energy equations are solved numerically for the combination of the conditions of restriction and swirling. The dimensionless quantities for the entropy generation, heat transfer and irreversibility are developed. They found that the irreversibility increases with increasing Prandtl number. The influence of swirling and Prandtl number on the non-dimensional entropy generation, heat transfer, irreversibility and the merit function were studied.

San and Jan [66] studied a second law analysis of a wet cross flow heat exchanger for various weather conditions. The effectiveness, exergy recovery factor and second-law efficiency of the wet heat exchanger are individually defined. The effects of lateral solid heat conduction on the effectiveness, exergy recovery factor and second-law efficiency are numerically investigated for various operating conditions. Two optimum design criteria, one for the maximum second-law efficiency and the other for the maximum exergy recovery factor, are obtained.

Mahmud et al. [67] analytically investigated the first and second law efficiency characteristics of fully developed non-Newtonian fluid flow and heat transfer inside a cylindrical annular space. Simplified governing equations in cylindrical coordinates are solved to obtain analytical expressions for dimensionless entropy generation number, irreversibility distribution ratio, and Bejan number as a function of flow governing and geometric parameters. Spatial distributions of local and average entropy generation rate and heat transfer irreversibility are presented graphically. The role of increased energy efficiency in achieving sustainable development is investigated by Rosen [68]. Specifically, the relations are examined between energy (and energy efficiency) and sustainable development.

Durmus [69] experimentally studied the heat transfer and exergy loss in a concentric heat exchanger with snail entrance. In their study, the effect of cut out conical turbulators, placed in a heat exchanger tube at constant outer surface temperature, on the heat transfer rates was investigated. The air was passed through the exchanger tube, the outer surface of which was heated with saturated water vapor. The experiments were conducted. Heat transfer, pressure loss and exergy analyses were made for the conditions with and without turbulators and compared to each other. Some empirical correlations expressing the results were also derived and discussed. Exergy analysis is shown to be an important tool for achieving sustainable development by Dincer and Rosen [70]. Several case studies highlight the insights revealed using exergy and exergetic aspects of sustainability. It is concluded that the potential usefulness of exergy analysis in addressing sustainability issues and solving environmental problems is substantial.

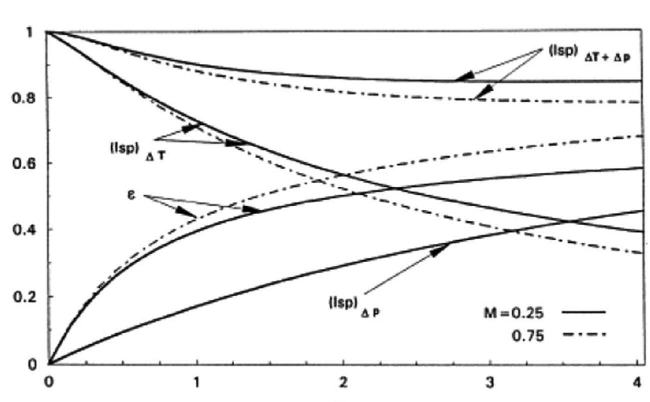

Fig. 10. Variations of different irreversibility components (thermal and frictional) and total irreversibility at two different dispersive wave propagation velocities (*M*), where I_{sp} is the specific irreversibility [64].

The effects on heat transfer, friction factor and dimensionless exergy loss were investigated experimentally by mounting helical (spring shaped) wires of different pitch in the inner pipe in a double pipe heat exchanger by Akpinar [71]. Dimensionless exergy loss rate expression is obtained by dividing exergy loss rate by the product of environmental temperature and minimum heat capacity. The experiments were performed for both parallel and counter current flow modes of the fluids at Reynolds numbers between 6500 and 13,000. An augmentation of up to 2.64 times in Nusselt number compared to the empty pipe was obtained in the helical system. The increase in friction factor was about 2.74 times that of the empty pipe, depending on Reynolds number and the pitch or helical number. An augmentation of up to 1.16 times in the dimensionless exergy loss compared to the empty pipe was obtained in the helical system. The exergy loss rate was defined as,

$$\dot{E}x_{loss} = \dot{W}_{max} - \dot{W}_{actual} \tag{64}$$

Dimensionless exergy loss rate expression was found as,

$$e = \frac{\dot{E}x_{loss}}{T_o C_{min}} \tag{65}$$

Heat transfer rates increased with decreasing pitch and with increasing helical number of the helical wires. The effectiveness values of the heat exchanger that used helical wires in the inner pipe was obtained up to 1.16 times compared to a heat exchanger with the empty pipe for the counter flow mode. Helical wires caused a considerable increase in pressure drop and friction factor. The dimensionless exergy loss and *Ntu* increased with the increase of helical number and decreased with the increase of pitch.

Gupta and Das [72] carried out second law analysis of cross flow heat exchangers in the presence of non-uniformity of flow. This non-uniformity is modeled with the help of axial dispersion model and takes into account the back mixing and flow maldistribution. They evaluated an analytical model for exergy destruction for the cross flow configuration. They carried out a wide range of study of the operating parameters and non-uniform flow on exergetic behavior of cross flow heat exchangers. The results clearly bring out not only the reason behind the maximum entropy paradox in heat exchangers but also the proper perspective of exergy destruction and the consequent optimization of cross flow heat exchangers from the second law viewpoint. The important feature in this analysis is the consideration of external irreversibility at the exit of the service fluid considering it to be thrown to the atmosphere while considering the process fluid at the exit of heat exchanger to be useful in terms of exergy utilization. The results indicate that this external destruction of exergy is the key to the so called 'maximum entropy paradox' related to a heat exchanger which is not isolated from the surroundings. When this external destruction is considered, the exergy destruction shows a monotonously decreasing trend with *Ntu*, which is consistent with the second law of thermodynamics. The results also indicate that how flow nonuniformity and maldistribution in the form of dispersive Peclet number influences not only the effectiveness of the heat exchanger but also the second

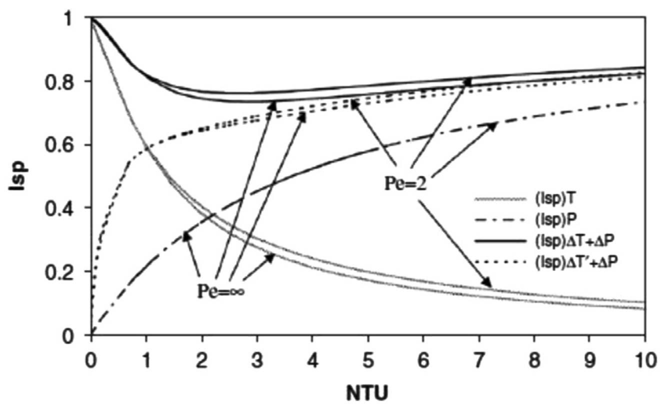

Fig. 11. Variations of different components of total specific irreversibility with *Ntu* at different Peclet numbers (Pe) [72].

law behavior. In view of the extensive application of crossflow heat exchangers in energy intensive areas such as automotive, cryogenics and refrigeration industry, the account of exergy destruction will give an important insight of utilization of energy resources related to these devices. The behavior of different components of total specific irreversibility namely due to heat transfer and pressure drop, with *Ntu* at different Peclet numbers is shown in Fig. 11.

Gupta et al. [73] defined the fractional exergy loss differently by dividing the irreversibility by maximum heat that can be transferred ideally so that one can get better understanding about the quantitative values of exergy loss in respect of maximum heat transferred. It is termed as irreversible number as follows:

$$\dot{I}_n = \frac{\dot{I}}{\dot{Q}_{max}} \tag{66}$$

The effect of these irreversibilities has been studied for both balanced and imbalanced flow of heat exchangers. It has been observed that the effect of the external thermal irreversibilities is more when the hot fluid is minimum capacity ($C_h = C_c < 1$) fluid and the contribution of the total external irreversibilities to the total thermal irreversibilities is only 2% when the value of C_h/C_c is 2. It has been shown that in the absence of pressure drop irreversibility, the performance of heat exchanger is less susceptible to the external irreversibilities for the given heat exchanger configuration in the case when the cold fluid is minimum capacity fluid. An attempt has been made to address the various external irreversibilities of the counter flow cryogenic heat exchangers in addition to the internal irreversibilities through second law analysis. They carried out this study because of the performance of highly effective heat exchangers was strongly dependent on these irreversibilities in low temperature applications. They observed that the influence of ambient heat-in-leak was different for the balanced and imbalanced counter flow high *Ntu* heat exchangers. Also, their study made it possible to compare the different irreversibilities for varying range of *Ntu* and analyze the effect of external irreversibilities on the performance of heat exchangers when either hot fluid or cold fluid was minimum capacity fluid.

Considerations related to size of application of exergy analysis are examined by Rosen [74]. The results can help guide exergy users to the most suitable manner of application for a given system and are expected to improve the usability of exergy analysis and increase its application. San [75] considered the exergy change rate in an ideal gas flow or an incompressible flow to analyze heat exchanger. Exergy change rate in an ideal gas flow or an incompressible flow can be divided into two types: a thermal exergy change rate and a mechanical exergy loss rate. They generalized the mechanical exergy loss rates in the two flows using a pressure-drop factor because the consumed mechanical exergy is usually more valuable than the recovered thermal exergy for heat exchangers using in waste heat recovery. The researcher proposed a weighing factor to modify the pressure-drop factor. Exergy recovery index was defined and it was expressed as a function of effectiveness, ratio of modified heat capacity rates, hot stream-to-dead-state temperature ratio, cold stream-to-dead-state temperature ratio and modified overall pressure-drop factor. A numerical example was used to derive the new results and compared with the existing methods of analysis.

A thermoeconomic performance optimization has been carried out for a single pass counter-flow heat exchanger model by Sahin et al. [76]. In the considered model, the irreversibilities due to heat transfer between the hot and cold stream are taken into account and other irreversibilities such as pressure drops and flow imbalance are ignored. The objective function is defined as the actual heat transfer rate per unit total cost considering lost exergy and investment costs. They considered the optimization of the actual heat transfer rate per unit total cost in order to account for both lost exergy and investment costs. The function to be optimized is defined as

$$F = \frac{\dot{Q}}{C_{LE} + C_I} \tag{67}$$

where C_{LE} and C_I refer to lost exergy and investment costs, respectively. The lost exergy cost of the heat exchanger is assumed to be proportional to the entropy generation rate and can be determined as,

$$C_{LE} = aT_o\dot{S}_{gen} \tag{68}$$

where the coefficient '*a*' is equal to the annual operation hours times price per unit monetary value of lost exergy. The investment cost of the heat exchanger is assumed to be proportional to the size of the heat exchanger. The size of the heat exchanger can be taken proportional to the total heat transfer area. Thus, the annual investment cost of the system is given as,

$$C_I = bA_s \tag{69}$$

where the investment cost proportionality coefficient '*b*' is equal to the capital recovery factor times investment cost per unit heat transfer area and A_s is the surface area of the system. The optimal performance and design parameters which maximize the objective function have been investigated. The effects of the technical and economical parameters on the general and optimal thermoeconomical performances have been discussed. The analysis and optimization presented will provide a basis for both determinations of the optimal performance parameters for real heat exchanger.

Energy and exergy analysis are defined to describe the performances of a new type of a polypropylene capillary heat exchanger operating as water–air and water–water heat exchanger by Hazami et al. [77]. Numerical and experimental investigations are carried out to analyze the thermal behavior with regard to mass flow rate and length effect in the ratio of the overall heat transfer coefficient and on energy and exergy effectiveness. Analytical analysis of unbalanced heat exchangers is carried out by Manjunath and Kaushik [78] to study the second law of thermodynamic performance parameter through second law efficiency by varying length-to-diameter ratio for counter flow and parallel flow configurations. In a single closed form expression, three important irreversibilities occurring in the heat exchangers namely, due to heat transfer, pressure drop, and imbalance between the mass flow streams are considered, which is not possible in first law of thermodynamic analysis. They used the rational efficiency equation for the unbalanced heat exchanger as

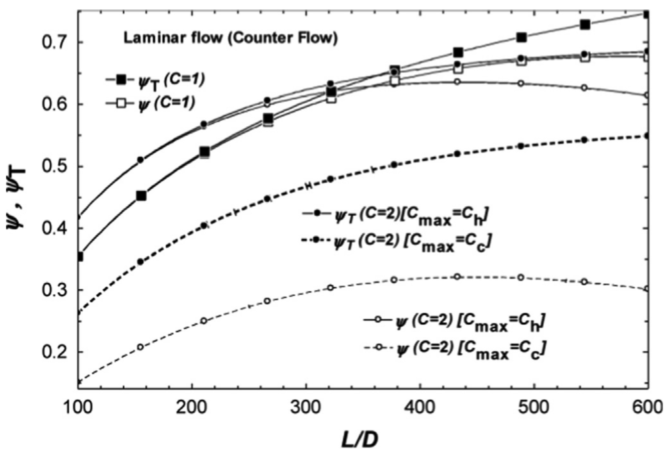

Fig. 12. Rational efficiency (ψ) and rational efficiency defined by heat transfer terms only (ψ_T) versus length-to-diameter ratio for laminar flow (counter flow) heat exchanger for different values of capacity ratio, C and for two cases, $C_{max}=C_h$ and $C_{max}=C_c$ [78].

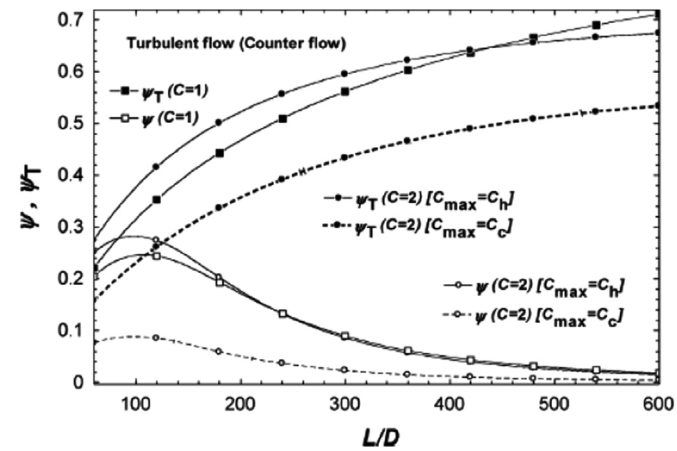

Fig. 13. Rational efficiency (ψ) and rational efficiency defined by heat transfer terms only (ψ_T) versus length-to-diameter ratio for turbulent flow (counter flow) heat exchanger for different values of capacity ratio, C and for two cases, $C_{max}=C_h$ and $C_{max}=C_c$ [78].

follows,

$$\psi = \frac{\dot{E}_{desired\ output}}{\dot{E}_{desired\ output} + \dot{I}} \tag{70}$$

From the consideration of the heat exchanger, they considered that the desired exergy output is the increase of the thermal component of exergy of the cold stream given as,

$$\dot{E}_{desired\ output} = \dot{E}_c = \dot{E}_{c,out} - \dot{E}_{c,in} \tag{71}$$

And $\dot{I}$ is the irreversiblitities occurring in the heat exchanger due heat transfer and pressure drop. The study is carried out by giving special influences on geometric characteristic like tube length-to-diameter dimensions; working condition like changing heat capacity ratio, changing the value of maximum heat capacity rate as hot stream and cold stream separately and fluid flow type i.e. laminar and turbulent flows for fully developed condition. Optimum heat exchanger geometrical dimension namely length-to-diameter ratio is obtained from the second law analysis corresponding to lower total entropy generation and higher second law efficiency. The results showed that second law analysis takes care of all the irreversibilities in the thermal design of heat exchanger as compared to first law analysis. The variations of second law efficiency that is rational efficiency with respect to length-to-diameter ratio for a counter flow heat exchanger is shown in Fig. 12 for laminar fluid flow and in Fig. 13 for turbulent fluid flow. This analysis provides a more realistic second law results and we can obtain an optimum value of L/D ratio for lower irreversibilities considering a single closed form expression.

3.3. Second law analysis of heat exchanger considering production irreversibility

Irreversibility associated with exergy of the material of construction of the heat exchanger is another way to analyze the performance of heat exchangers. Aceves-Saborio et al. [7] have carried out irreversibility minimization analysis applied to heat exchangers. Here the irreversibility minimization method of heat exchangers optimization is extended to include a term to account for the exergy of the material of construction of heat exchanger. They considered irreversibility due to material of construction and derived relationship between exergetic efficiency and effectiveness. This exergetic efficiency gives physically more realistic values than the usual expressions. Cumulative exergy destruction associated with the production of material and manufacturing of heat exchanger is due to destructions occurring during following processes: (1) primary process used to convert natural resources like ores to useful form like metallic slabs. (2) Secondary process that consists of producing tubes, sheets, etc. from primary material obtained from primary process. (3) Manufacturing process, this is used to fabricate the end product like tube bending, welding, etc. They defined irreversibility rate assignable to the materials of the heat exchanger as follows,

$$\dot{I}_m = \frac{E_m}{t} \tag{72}$$

where E_m is the exergy of the materials and t is application life of the heat exchanger (total operating time of heat exchanger in its life time). They took into account the irreversibility associated with the use of the materials, but did not include the irreversibility due to the pressure drops.

Cornelissen and Hirs [12] introduced the concepts of an exergetic life cycle analysis and the minimization of life cycle irreversibility, which can be applied for the optimal design of a component, where there is a trade-off between exergy saving during operation and irreversibility during construction. The analysis has been carried out for the optimal design of a heat exchanger in a district heating system. The optimization takes into account irreversibilities due to frictional pressure drops, the temperature difference between the hot and cold stream and irreversibilities due to the production of the materials and the construction of the heat exchanger. Irreversibilities associated with production of materials are included along with the general irreversiblities such as due to heat transfer and pressure drop in the heat exchanger. They showed that the concepts of an exergetic life cycle analysis, the minimization of life cycle irreversibility, can be applied for the optimal design of a component, where there is a trade-off between exergy saving during operation and irreversibility during construction. Irreversibility associated with the use of material and manufacturing of heat exchanger was provided as,

$$\dot{I} = \frac{MC_{CU}}{t} \tag{73}$$

where M is the mass of the equipment, C_{CU} is the cumulative exergy destruction due to material and manufacturing of heat exchanger and t is the operating time of heat exchanger during its life cycle. As an example of this type of heat exchanger a water-to-water heat exchanger in a district heating system has been analyzed. The influence of the configuration of the heating system,

including the energy conversion, on the optimization of the heat exchangers has been shown.

The cumulative exergy of a heat exchanger production process is studied considering cumulative exergy analysis by Xiao et al. [79]. The cumulative exergy includes the cumulative exergy of raw materials, the electricity consumption, and the contribution of the equipment used in the production process. Because the production of heat exchangers involves mainly machining processes, the energy consumption is relatively small. As a result, the cumulative exergy of raw materials is the major component of the total value. Furthermore, based on the cumulative exergy consumption theory, a heat exchange process is optimized. The cumulative exergy approach is applied to air conditioning system and presented by Jing et al. [80]. The results are different from those of the exergy analysis, indicating that cumulative exergy analysis is an effective method to quantitatively evaluate different air conditioning systems according to resource utilization.

3.4. Second law analysis of two-phase flow heat exchangers

The second law analysis of two-phase flow heat exchangers has received considerable attention by several investigators. London and Shah [81] discussed the thermodynamic irreversibilities that exist in any real system such as condenser, which is a phase change heat exchanger. They demonstrated a method to attach monetary values to component irreversibilities generated in the condenser of a power plant with the purpose of developing various tradeoff factors. Heat transfer irreversibility is given as,

$$\left.\frac{\dot{I}}{q}\right|_{q} = \left(1-\frac{T_{C,lm}}{T_{H,lm}}\right)\frac{T_o}{T_{C,lm}} \tag{74}$$

where q is heat transfer rate, $T_{C,lm}$ and $T_{H,lm}$ is the cold and hot fluid logarithmic mean temperatures. The above result is applicable to single-component, two-phase condensation, by treating $T_{H,lm}$ as identical to the constant condensation temperature, or to two phase evaporation, by treating $T_{C,lm}$ as identical to the constant evaporation temperature. Pressure drop irreversibility equation is derived in general for condensation or evaporation as follows,

$$\left.\frac{\dot{I}}{q}\right|_{\Delta P} = \frac{T_o}{T_{ave}}\frac{\Delta P/\rho_v}{2h_{lg}} \tag{75}$$

where T_{ave} is the average temperature, ρ_v is the density of saturated vapor and h_{lg} is the specific enthalpy change on vaporization. Their methodology, in the hands of the heat exchanger designer, allowed an interaction with the system designer to gain insights into the trade-offs allowed between the thermodynamic irreversibilities of flow friction, heat transfer, heat leakage, and mixing. Their methodology started with the recognition of the appropriate individual irreversibilities. Then, it related the individual costs to system rating and energy penalties by thermodynamic arguments. The analysis loop was closed by considerations related to reduction of the individual irreversibilities in a cost-effective way. On the other hand, the exergy analysis provided an answer for the overall costs of the collective irreversibilities. This did not provide the engineer with the insight needed to minimize the individual irreversibilities in a cost-effective manner.

Zubair et al. [82] presented a closed form analytical method for the second law based thermoeconomic optimization of two-phase exchangers used as condensers or evaporators. Due to finite temperature difference heat transfer and pressure drops, they proposed the concept of “internal economy” as a means of estimating the economic value of entropy generated, thus permitting the engineer to trade the cost of entropy generation in the heat exchanger against its capital expenditure. Results were presented in terms of the optimum heat exchanger area as a function of the exit/inlet temperature ratio of the coolant, unit cost of energy dissipated and the optimum overall heat transfer coefficient. They presented the analysis of a water-cooled condenser and an air-cooled evaporator with supporting numerical examples that were based on the thermoeconomic optimization procedure of this study.

Second law analysis of two-phase flow heat exchangers were also conducted by Lau et al. [83]. They identified the essential design parameters for determining the optimum configuration of an air-cooled condenser. For a power plant operating on a Rankine cycle, expressions for (i) the minimum frontal area, (ii) the minimum heat transfer area, and for (iii) the maximum net cycle efficiency, with respect to the condenser temperature and the cooling air velocity are derived. The analyses are carried out with the assumption that the exit temperature of the cooling air is equal to the condenser temperature. All resulting equations are presented in dimensionless form so that they are applicable to any power cycle with a gas-cooled condenser. The general form of entropy generation equation for two phase equation is provided by Bejan [2] as defined earlier in Eq. (27).

A straightforward, entropy-based method for evaluating air-side performance of a condenser in a vapor-compression system was proposed by Dejong et al. [84]. They concluded that the effects of the heat exchanger design on the system can cause significant exergy destruction and must be considered by any good performance measure. The proposed methods were applied to the evaluation of a condenser in a vapor-compression system. The condenser example is discussed in detail, to explore design tradeoffs.

The optimum thermodynamic match between two streams at different temperatures is determined by maximizing the power generation (or minimizing the entropy generation) associated solely with the stream-to-stream interaction as provided by Vargas and Bejan [85]. Each stream experiences a change of phase. The optimum is marked by an optimal ratio between the stream mass flow rates, and an optimal ratio between the two heat exchanger sizes when the total heat transfer area is fixed. The sensitivity of the optimum relative to the various physical parameters of the two-stream arrangement is documented systematically. They showed that the optimum is robust relative to changes in several parameters such as the distribution of heat transfer coefficient along the hot-end heat exchanger, and the model used for the thermodynamic behavior of stream. They defined second law efficiency as,

$$\eta_{II} = \dot{W}/(\dot{m}_h e_{x,h}) \tag{76}$$

where W is the power, m_h is the mass flow rate of hot stream and $e_{x,h}$ is specific exergy of hot stream.

Lin et al. [86] carried out the condensation process of saturated vapor flowing through horizontal cooling tubes using second law analysis. For single and multiple tubes, the effect of Reynolds number that minimizes the entropy generation rate over the parametric range has been investigated. Variations of total entropy generation, due to heat transfer and due to pressure drop with respect to mass velocity in condensation process of saturated vapor flowing through horizontal cooling tubes is shown in Fig. 14 for different values of inlet quality of two phase fluid. Can et al. [87] carried out exergoeconomic analysis of condenser type heat exchangers to find the optimum working conditions. Theoretical fundamentals and practical applications of exergoeconomic analysis are presented, specifically in two phase heat exchangers and generally for energy transportation systems. Annual operation hour and unit price of electrical energy are taken into account for determination of the annual operation expenses. Investment expenses are obtained according to the variation of heat capacity

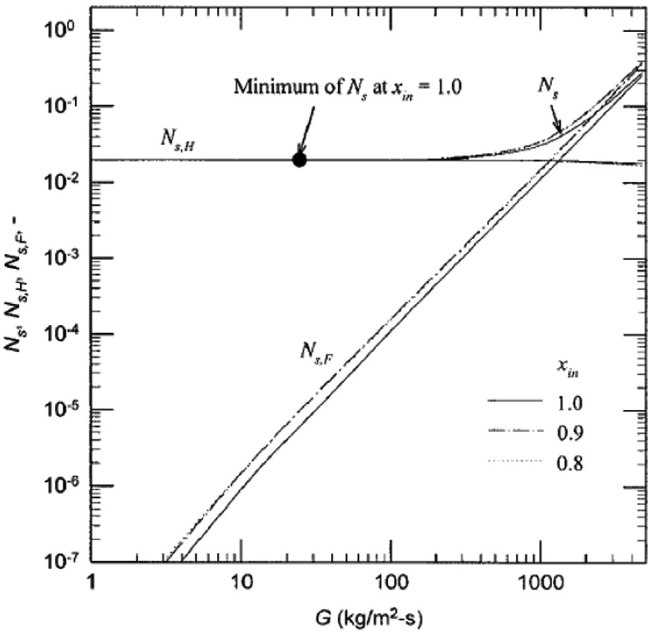

Fig. 14. Variations of total entropy generation number (N_s), due to heat transfer ($N_{s,H}$) and due to pressure drop ($N_{s,F}$) with respect to mass velocity in condensation process of saturated vapor flowing through horizontal cooling tubes, where x_{in} is the quality of two phase fluid at inlet [86].

rate and logarithmic mean temperature difference and also heat exchanger dimension determined for each situation. Their analysysis resulted in determining the effective parameters for the most appropriate exergy losses together with operating conditions and in finding the optimum working points for the condenser type heat exchangers.

Dentice d'Accadia et al. [88] carried out an exergoeconomic analysis to search for the optimal design of a condenser to be used in a conventional vapour-compression heat pump. They presented how the use of simple thermodynamic and thermoeconomic optimization methodologies in refrigeration could contribute to determining the correct design of new equipment. Dentice d'Accadia and Vanoli [89] proposed the design optimization of a heat exchanger using a thermoeconomic approach. The investigation is referred to the tube-in-tube condenser of a conventional vapour-compression heat pump, with a two-phase refrigerant flowing in the inner tube and a single-phase fluid flowing in the annulus. They discussed case for a commercial heat exchanger and the design improvements needed to obtain a cost-optimal configuration are investigated. Lingen et al. [90] showed the design of a refrigerator or an air-conditioning plant can be optimized by distributing the total heat-exchanger surface-area between the two heat-exchangers in the plant. Entropy production and energy availability destruction have been calculated for film condensation heat transfer by Adeyinka and Naterer [91]. Their optimization study suggests that entropy production can serve as a useful parameter in the thermal design of a two phase system.

A multi-objective optimization procedure is implemented to find optimal design values for design variables for heat exchangers of refrigerator by Gholap and Khan [92]. The analysis presented an effective method for finding a set of the best trade-off design solutions for heat exchangers in the presence of two competing objectives for multiple design variables using a detailed refrigeration-cycle simulation model. Entropy generation of vapor condensation in the presence of a non condensable gas in a shell and tube condenser was carried out by Haseli et al. [93]. The thermal irreversibility of heat exchange between the bulk fluid and condensate, as well as heat exchange between the condensate and coolant, are taken into account. The relevant entropy generation numbers are established and shown as functions of the non-dimensional temperature and heat transfer coefficient. Haseli et al. [94] carried out the exergy destruction and exergy efficiency analysis for condensation of a vapor in a shell and tube condenser. They focused on evaluation of the optimum cooling water temperature during condensation of saturated water vapor within a shell and tube condenser, through minimization of exergy destruction.

Optimal configuration of cross flow plate finned tube condenser based on the second law of thermodynamics was obtained by Saechan and Wongwises [95]. Results from the mathematical model showed that entropy generation number decreases with increasing fin pitch and decreasing number of rows and tube diameter. The harmony search algorithm was employed to find optimum design configuration of air cooled condensers by Doodman et al. [96]. The algorithm ability was demonstrated using a case study and the performance was compared with genetic algorithm. Haseli et al. [97] investigated a *TEMA "E"* shell and tube condenser through exergy efficiency as a potential parameter for performance assessment. Exergy analysis of condensation of pure vapor in a mixture of non-condensing gas such as industrial condenser with a steam–air mixture and cooling water as working fluid is presented.

Kaushik and Manjunath [98] analyzed a condenser based on second law analysis by using new heat transfer and pressure drop model based on two phase flow regimes namely annular, stratified wavy and fully stratified patterns. For each type of flow patterns, the behavior of entropy generation number due heat transfer and pressure drop were studied by varying different input parameters of two phase fluid such as diameter of two phase tube, mass velocity, difference between saturation and tube wall temperature, quality and saturation temperature. They concluded that for proper thermal designing of condenser, we need to consider different flow regimes analysis for calculating overall heat transfer coefficient and pressure drop for different input values. Also, we need to consider second law analysis aspect for proper selection of design parameters for different applications to reduce irreversibilities. Part of their work is reproduced in Figs. 15 and 16, which is entropy generation number due to heat transfer and pressure drop with respect to diameter in condensation process of refrigerant for two phase fluid patterns namely annular, stratified wavy and fully stratified flows. As the value of d is increased, the N_{sh} value which will be maximum for lower value of d starts decreasing and reaches a minimum value. This will be useful to select the value

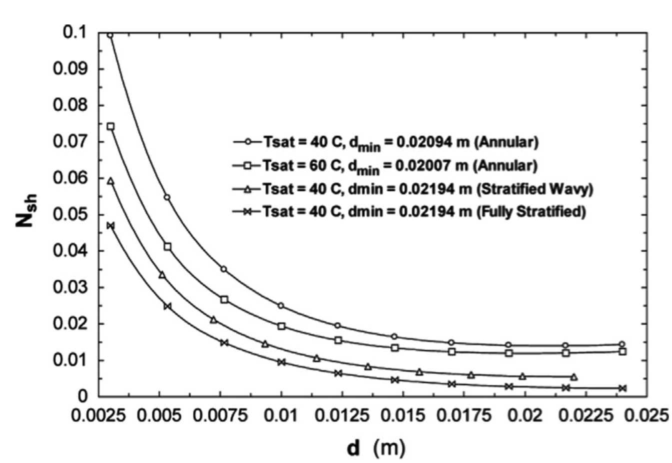

Fig. 15. Variations of entropy generation number due to heat transfer (N_{sh}) with respect to diameter in condensation process of refrigerant for two phase fluid patterns namely annular, stratified wavy and fully stratified flows [98].

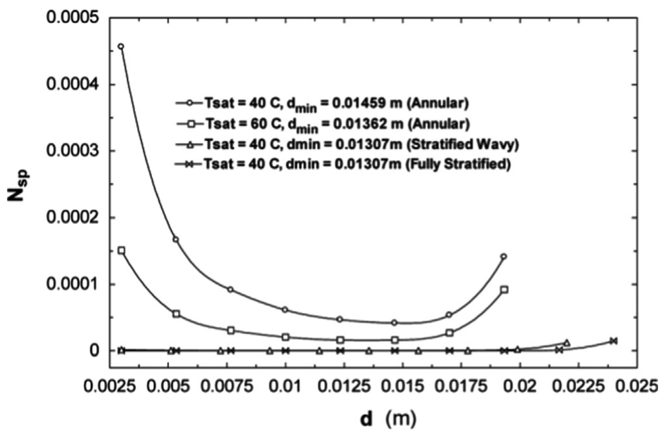

Fig. 16. Variations of entropy generation number due to pressure drop (N_{sp}) with respect to diameter in condensation process of refrigerant for two phase fluid patterns namely annular, stratified wavy and fully stratified flows [98].

of diameter for minimum N_{sh}. Same result follows for the behavior of N_{sp} versus d.

Ye and Lee [99] studied a numerical model that is applicable to fin-and-tube condensers with complex refrigerant circuitry which can be used to describe the exergy loss on the refrigerant side. A methodology was developed for refrigerant circuitry design based on entropy generation minimization. The resulting refrigerant circuit design enhanced heat transfer performance and lowered entropy generation in comparison to simple refrigerant circuitries. Entropy generation and thermoeconomic analysis of air cooled natural draft wire-and-tube condenser commonly used in domestic refrigerator is carried out by Kaushik and Manjunath [100]. The analytical study is carried out by varying geometrical parameters of the condenser namely tube outer diameter, wire diameter, number of wire pairs, number of tube rows, tube pitch, wire pitch, and refrigerant (*R134a*) properties namely mass velocity, saturation temperature and dryness fraction. Geometrical dimensions and operating parameters can be obtained for better performance of wire-and-tube condenser based on second law and thermoeconomic analysis for the data considered. Thermoeconomic analysis not only takes care of second law based analysis, but also attaching economics to the irreversibilities occurring in the process.

Differences between the analyses using entropy generation over exergy as evaluation parameter are as follows [14]:

a. The entropy measure ties in directly with the inequality sign of the second law. In this sense, as it is more basic, there should be general agreement on its usage.
b. It is operationally more convenient to account for entropy as a single extensive function of state than to account for the availability functions ($h–T_o s$) for the flow terms and ($u+P_o v-T_o s$) for the storage terms, involving a multiplicity of both extensive and intensive state functions.
c. The conversion of the entropy measure to the availability measure can be achieved readily using relations of equation $\dot{I}=T_o\dot{S}_{gen}$.
d. In some situations it is more reasonable to use a temperature weighing factor other than T_o which makes analysis more complicated as in the case of exergy analysis.

3.5. Constructal theory applied to heat exchangers

The constructal law was proposed in the year 1996 as a summary of all design generation and evolution phenomena in nature by Bejan [4]. The constructal law represents three steps towards making "design in nature" a concept and law-based domain in science given by Bejan [6]:

a. Life is flow: all flow systems are live systems, the animate and the inanimate.
b. Design generation and evolution is a phenomenon of physics.
c. Designs have the universal tendency to evolve in a certain direction in time.

In the big picture, all the mating and morphing flows on the largest system that surrounds us, the Earth itself, evolve to enhance global flow. For example, trees and other forms of vegetation that move moisture from the ground to the air are components of the larger global system, including forests, river basins and weather patterns, that have the tendency to equilibrate all the moisture on Earth. The constructal law states that every flow system is destined to remain imperfect. The direction of design evolution is toward distributing the imperfections of the system, such that the "whole" flows easier (e.g., river basin, animal body, human vehicle). Evolution never ends. Optimality statements (minimum, maximum, optimum, end design, destiny), have no place in constructal theory. Nature does not move toward an optimal end design. The natural phenomena are not the elimination but the distribution (better and better over time) of imperfection. The distribution of imperfection generates the geometry (shape, structure) of the system as provided in Bejan and Lorente [101]. For example, in point-area and point-volume flows, constructal theory predicts tree architectures, such flows displaying at least two regimes: one highly resistive and one with lower resistivity. The constructal-law tendency manifests itself at every scale.

Constructal theory proceeds in time against empiricism or copying from nature and its concept is explained in Fig. 17. Constructal theory proclaims the oneness of natural and engineered flow configuration generation phenomena. Nature and Engineering can be contemplated together in two ways, empirically and theoretically. In empiricism, the observing and copying from nature come first, and this serves as basis for modeling, description, and bio-inspired engineering. In constructal theory the thought process goes against the time arrow of empiricism: first, the constructal law is invoked, and from this thought the flow architecture is deduced. Only later is the theoretical configuration compared with a natural configuration, and the agreement between the two validates the constructal law.

Classical thermodynamics versus constructal thermodynamics in a broad way can be understood as provided in Bejan and Lorente [102] as follows. Thermodynamics rests on two laws. Both

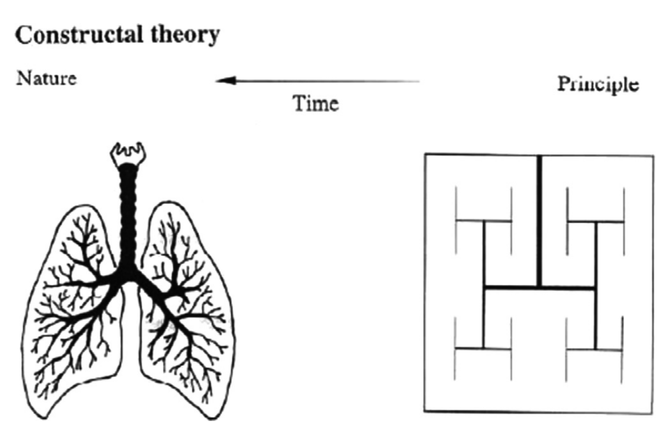

Fig. 17. Concept of constructal theory. Constructal theory proclaims the oneness of natural and engineered flow configuration generation phenomena [101].

are first principles: (a) the first law commands the conservation of energy; (b) the second law summarizes the tendency of all currents to flow from high (temperature, pressure) to low. These two laws are about systems in the most general sense, viewed as black boxes, without shape and structure. The two laws of thermodynamics do not account for nature completely. Nature is not made of black boxes. Nature's boxes are filled with configurations – even the fact that they have names (rivers, blood vessels) is due to their appearance, pattern, or design. Whereas the second law commands that things should flow from high to low, the constructal law commands that they should flow in configurations that flow more and more easily over time.

Applications of constructal theory are

a. River basin design.
b. The distribution of hot water in cities.
c. Cooling of electronic chips.
d. Cooling of IC engines.
e. Cooling of fuel cells.
f. The entire architecture of vegetation: roots, trunks, canopies, branches, leaves, and the forest.
g. The scaling law of all animal locomotion (running, flying, swimming): speeds, frequencies, forces and the work spent per unit of mass moved and distance traveled.
h. The relationship between breathing and heart beating times and body size.
i. The human bronchial tree with 23 levels of bifurcation.
j. Heat transfer and fluid flow systems.

Constructal theory has been used to analyze heat exchanger by several investigators as follows. Constructal theory is also used to optimize the performance of thermo-fluid flow systems by generating geometry and flow structure, and to explain natural self-organization and self-optimization. Constructal theory is a principle-based method of constructing machines, which attain optimally their objective as discussed in [103]. Constructal theory holds that every flow system exists with purpose (or objective, function). In nature, flows occur over a wide range of scales with the purpose of reducing the existing gradients (temperature, pressure, etc.). In engineering and living structures heat and mass flows occur for the same reason, and by dissipating minimum exergy they reduce the food or fuel requirement, and make all such systems (animals, and “man+machine” species) more “fit,” i.e., better survivors. They “flow” better and better, internally and over the surface of the earth. The economics of the constructal theory of generation of shape and structure in natural flow systems that connect one point to a finite size area or volume is described by Bejan et al. [104]. The constructal theory, as extended in this analysis, unites the naturally-organized flow structures that occur spontaneously over a vast territory, from geophysics to biology and economics.

Bejan [105] described the constructal route to the conceptual design of a two-stream heat exchanger with maximal heat transfer rate per unit volume. The flow structure has multiple scales. The smallest (elemental) scale consists of parallel-plates channels the length of which matches the thermal entrance length of the small stream that flows through the channel. This feature has two advantages: it eliminates the longitudinal temperature increase (flow thermal resistance) that would occur in fully developed laminar flow, and it doubles the heat transfer coefficient associated with fully developed laminar flow. The elemental channels of hot fluid are placed in cross flow with elemental channels of cold fluid. The elemental channel pairs are assembled into sequentially larger flow structures (first construct, second construct, etc.), which have the purpose of installing (spreading) the elemental heat transfer as uniformly as possible throughout the heat exchanger volume. At length scales greater than the elemental, the streams of hot and cold fluid are arranged in counter flow. Each stream bathes the heat exchanger volume as two trees joined canopy to canopy. One tree spreads the stream throughout the volume (like a river delta), while the other tree collects the same stream (like a river basin). It is shown that the spacings of the elemental and first-construct channels can be optimized such that the overall pumping power required by the construct is minimal. They concluded the advantages of the proposed tree-like (vascularized) heat exchanger structure over the use of parallel small-scale channels with fully developed laminar flow. Dendritic heat exchangers have triggered a vascularization race not only in heat exchangers but also in electronics cooling, fuel cells architecture conceptualization and fluid distribution and collection in general.

Wechsatol et al. [106] investigated the constructal optimization of fluid flow problem between the center and various equidistant points on the circumference by taking minimum flow resistance as optimization objective in detail. The optimization parameters included the number of channels at the center, the order of the connection pipes between the center point and the points on the circumference, and the optimal structure was two sub-branches when the last-order pipe was divided into the new-order pipes. The smallest length scale of the flow structure is fixed (d), and represents the distance between two flow ports on the circular perimeter. The paper documents a large number of optimized dendritic flow structures that occupy a disc-shaped area of radius R. The flow is laminar and fully developed in every tube. The complexity of each structure is indicated by the number of ducts (n_0) that reach the central point, the number of levels of confluence or branching between the center and the perimeter, and the number of branches or tributaries (e.g., doubling vs. tripling) at each level. The results show that as $R=d$ increases and the overall size of the structure grow, the best performance is provided by increasingly more complex structures. The transition from one level of complexity to the next, higher one is abrupt. Generally, the use of fewer channels is better, e.g., using two branches at one point is better than using three branches. As the best designs become more complex, the difference between optimized competitors becomes small. These results emphasize the robustness of optimized tree-shaped networks for fluid flow.

Chen and Cheng [107] proposed a fractal tree-like micro-channel net heat sink for cooling of electronic chips. The micro-channel net was designed to have a top and a bottom circulation pattern in a wafer. The study showed that this type of heat sink had better heat transfer characteristics and required less pumping power than traditional parallel nets. Ordonez and Bejan [108] showed that the sizes (weights) of heat and fluid flow systems that function on board vehicles such as aircraft can be derived from the maximization of overall system performance. The total weight of the aircraft dictates its fuel requirement. Components, power plants and refrigeration plants operate less irreversibly when they are larger. Less irreversibility means less fuel needed for their operation. On the other hand, larger sizes add more to the mass of the aircraft and to the total fuel requirement. This tradeoff pinpoints optimal sizes. The principle is illustrated based on three examples: a power plant the size of which is represented by a heat exchanger, a counter flow heat exchanger without fluid flow irreversibility, and a counter flow heat exchanger with heat transfer and fluid flow irreversibilities. The size optimization principle is applicable to the organs of all flow systems, engineered (e.g., vehicles) and natural (e.g., animals).

Bonjour et al. [109] investigated heat exchange process with counter flows in the two coaxial pipes, and optimized the fin set between the two pipe walls. For the analysis, example of the coaxial two-stream heat exchanger with flow through a porous

bed in the annular space was considered. It is shown that the constraints force the design toward heat exchangers with finite axial length, where additional improvements are derived from installing high-conductivity fins across the porous bed. The maximization of global performance is achieved through the optimization of the configuration of plate fins. Configurations with radial fins are optimized analytically and numerically. Configurations with branched fins are optimized numerically. It is shown that the best configuration (radial vs. branched) depends on the size of the heat exchanger cross-section. When the size is small, the best is the radial pattern. When the size exceeds a certain threshold, the best configuration is the optimized branched tree of fins.

Zamfirescu and Bejan [110] investigated constructal tree-shaped two-phase flow for cooling a surface. They studied the optimal structure of the phase change with convective heat transfer. The system is a surface with uniform heating per unit area, which is cooled by a network with evaporating two-phase flow. Illustrations are based on the design of the cooling network for a skating rink. The flow structure is optimized as a sequence of building blocks, which starts with the smallest (elemental volume of fixed size), and continues with assemblies of stepwise larger sizes (first construct, second construct, etc.). The optimized flow network is tree shaped. Three features of the elemental volume are optimized: the cross-sectional shape, the elemental tube diameter, and the shape of the elemental area viewed from above. The tree that emerges at larger scales is optimized for minimal amount of header material and fixed pressure drop. The optimal number of constituents in each new (larger) construct decreases as the size and complexity of the construct increase. Constructs of various levels of complexity compete: the paper shows how to select the optimal flow structure subject to fixed size (cooled surface), pressure drop and amount of header material.

da Silva et al. [111] described the conceptual design and performance of balanced two-stream counter flow heat exchangers, in which each stream flows as a tree network through its allotted space. The two trees in counter flow are like two palms pressed against each other. They developed the relationships between effectiveness and number of heat transfer units for several tree-counter flow configurations: (i) constructal dichotomous trees covering uniformly a rectangular area, (ii) trees on a disk-shaped area, and (iii) trees on a square-shaped area. The paper reports the formula for the number of heat transfer units in each configuration. Unlike in counter flows formed by two parallel streams, in which the longitudinal temperature gradient is constant, in the counter flow formed by two trees the longitudinal temperature gradient is steeper as one approaches the periphery of the tree canopy.

Following the procedure provided in da Silva et al. [111], referring to Fig. 18, the counter flow heat exchanger has two identical trees, one for the hot fluid and the other for the cold fluid. The two trees match perfectly to each other, so that one tube of the hot tree is parallel to and in perfect thermal contact with the corresponding tube of the cold tree. Insulations are provided as shown in the figure to prevent external heat loss.

Each of the trees is made up of tubes of $(n+1)$ sizes. Each tube has the length L_i and internal diameter D_i, where $i=0,1,\ldots,n$. Tube lengths double after two consecutive construction steps. For entire tree structure the length-doubling formula can be expressed approximately for laminar flow as,

$$\frac{L_{i+1}}{L_i}=2^{1/2} \tag{77}$$

As we considered pairing at every construction level, the tube numbers and flow rates are expressed as,

$$n_i=2^{n-i} \text{ and } \dot{m}_i=2^i\dot{m}_o$$

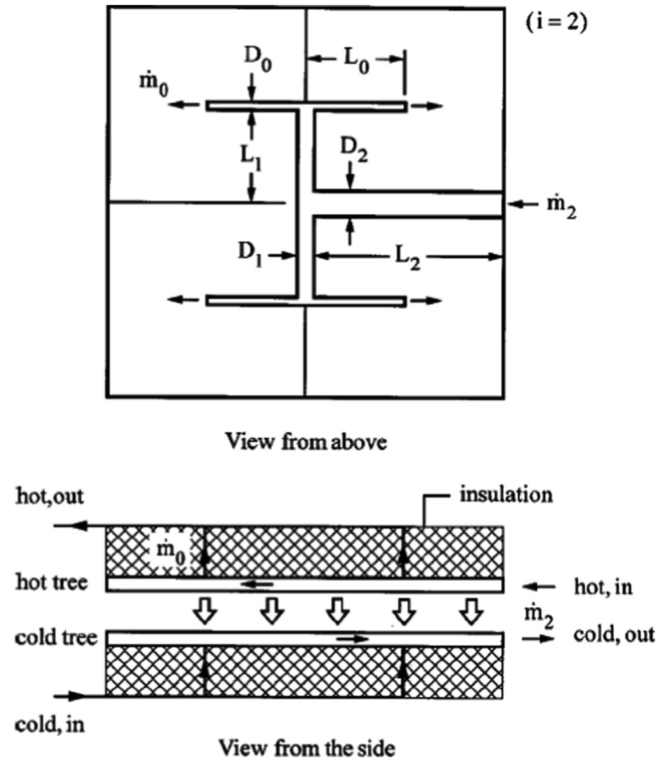

Fig. 18. Constructal counter flow of tree shaped streams distributed over a square area [111].

For n level of construction stages, at the end, stream mass flow rate will be $\dot{m}_n$ which is equal to $2^n\dot{m}_o$ flows through nth construct. Inner diameter of tube gets incremented for laminar flow as,

$$\frac{D_{i+1}}{D_i}=2^{1/3} \tag{78}$$

Some of the assumptions made while carrying out the analysis as provided in [111] are: (1) the flow is fully developed laminar, (2) the pressure drops are mainly due to friction along the straight cross section of the heat exchanger, (3) neglecting the local pressure drops associated in the joints of tubes, (4) one tube of hot stream tree is right next to its counterpart in the cold stream tree and has excellent thermal contact, (5) same type of fluid is flowing in both the streams, (6) the heat exchanger is balanced one.

The stream-to-stream heat transfer rate of the exchanger is given as,

$$Q_i=U_i\pi D_i L_i \Delta T_i \tag{79}$$

where ΔT_i is the temperature difference between the hot and cold streams which remains constant throughout the balanced heat exchanger. Neglecting the thickness of the tubes and considering same type of fluid flowing in the both the tubes, the overall heat transfer coefficient is given as [111],

$$\frac{1}{U_i}=\frac{1}{h_i}+\frac{1}{h_i}$$

The heat transfer coefficient is related to Nusselt number as,

$$h_i=\frac{k}{D_i}Nu$$

Ntu expression is obtained by considering stream-to-stream heat transfer and enthalpy difference heat transfer as provided

in [111] for constructal heat exchangers as follows,

$$Ntu_i = \frac{\pi k Nu L_i}{2\dot{m}_i c_p} \tag{80}$$

Performing the summation for whole heat exchanger, we will obtain the global (tree) number of heat transfer units as,

$$\frac{\Delta T_x}{\Delta T} = \frac{\pi k Nu L_0}{\dot{m}_n c_p} 2^{n-1} S_1 = Ntu \tag{81}$$

where ΔT_x is the overall temperature difference between the root (inlet or outlet) and the canopy points (outlets or inlets) and ΔT is the stream-to-stream temperature difference, which remains same throughout the exchanger. Where S_1 is a sum that depends on the complexity of the tree structure (n),

$$S_1 = (1-2^{-(n+1)/2})/(1-2^{1/2}) \tag{82}$$

Global (tree) number of heat transfer units can be derived in terms of tree complexity, n and dimensionless mass flow rate, M as,

$$Ntu = \frac{2^{((n/2)-2)} S_1}{M} \tag{83}$$

where dimensionless mass flow rate is given as,

$$M = \frac{\dot{m} c_p}{\pi k Nu A^{1/2}} \tag{84}$$

Global effectiveness of the balanced tree counter flow heat exchanger is given as,

$$\varepsilon = \frac{\Delta T_x}{\Delta T_x + \Delta T} = \frac{Ntu}{Ntu+1} \tag{85}$$

The global thermal resistance of the heat exchanger is given as [111],

$$R_t = \frac{1}{\dot{m}_n c_p \varepsilon} \tag{86}$$

The dimensionless thermal resistance is given as,

$$\tilde{R}_t = \frac{1}{M\varepsilon} \tag{87}$$

where M is the dimensionless mass flow rate given by Eq. (84).

Making the assumption that every tube is slender and the flow is in the Poiseuille regime: the pressure drops are mainly due to friction along the straight sections of the network. In other words, neglecting the local pressure drops associated with joining two tubes together. The pressure drop along one tube of type i is [111],

$$\Delta P_i = \frac{128}{\pi} \nu \dot{m}_i \frac{L_i}{D_i^4} \tag{88}$$

Performing the summation, we will obtain the global (tree) pressure drop as,

$$\Delta P = \frac{128}{\pi} \nu \dot{m}_n \frac{L_o}{D_o^4} 2^{-n} S_2 \tag{89}$$

where, $S_2 = (2^{(n+1)/6}-1)/(2^{(1/6)}-1)$.

The pumping power required to force total mass flow rate through one of the trees is,

$$\dot{W} = \dot{m}_n \frac{\Delta P}{\rho} \tag{90}$$

The dimensionless power requirement is obtained with the help of Eqs. (89) and (90) as,

$$\tilde{W} = \pi^3 2^{-n/2} S_2^3 M^2 \tag{91}$$

where M is the dimensionless mass flow rate given by Eq. (84).

The sample part of investigations carried out by [111] is shown in Fig. 19. Here we have three classes of two-stream counter flow heat exchangers, each having the same external size and internal size (total flow volume). The rightmost curves belong to the configuration consisting of two adjacent tubes which are defined in [111]. The dashed line corresponds to two disk-shaped sheets of fluid in counter flow. The leftmost curves are for counter flows made of two trees touching like two palms pressed against each other. The curve marked with circles corresponds to the configuration of disc shaped heat exchanger where the two sides of the heat transfer surface are point-circle and circle-point trees of the type optimized. The curve marked with squares is for two trees mounted on a square heat transfer surface. These heat exchangers provide an example of the evolution of the flow architecture in the two-objective design space.

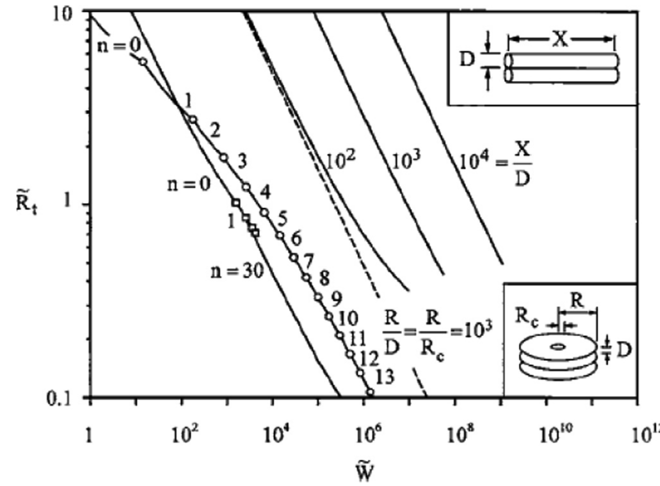

Fig. 19. The evolution of thermofluid performance of architectures for two stream counter flow heat exchangers, from right to left, two adjacent tubes, two radial flow sheets, and two trees. Where $\tilde{R}$ is the dimensionless thermal resistance, $\hat{W}$ is the dimensionless power requirement and X/D is the slenderness ratio [111].

The application of constructal heat exchangers to devices with maximal transport density is discussed, e.g., electronics cooling, fuel cell architectures, etc. Muzychka et al. [112] studied constructal design of force convection cooled microchannel heat sinks and heat exchangers. They investigated the heat transfer performances of different shapes of micro-channels involving parallel plates, rectangles, squares, ellipses, rounds, triangles and polygons.

Raja et al. [113] studied the constructal optimization problem of heat exchangers by using air and water as the heat exchange mediums, and performed numerical calculations for the fluid flow and heat transfer problems. Result of forced convection heat transfer in the fully developed flow is compared with the thermally developing flow. It was found that the heat transfer coefficients were higher due to small channel spacing and developing laminar flow. Also, there was sudden increase in heat transfer coefficient at the exit region of channels due to axial conduction. The increase in effectiveness was found to be 10% for the first construct. It was concluded that in the dendritic constructal heat exchanger higher effectiveness is achievable compared to conventional cross flow heat exchanger. Zimparov et al. [114] used the concepts of Silva et al. [111] to analyze performance of balanced two-stream parallel flow heat exchangers. The performance of the parallel flow configurations is compared with the performance of counter flow configurations. The future use of constructal heat exchangers in devices with maximal heat transport density is also proposed.

da Silva et al. [115] conducted experiments on heat exchanger device with tree-shaped flow channels in a disc area to validate their theory. The tree structure is the same on both sides of the heat exchanger: they have three channels reaching/leaving the center, and three branching levels. On the hot side, fluid is pumped

from the center to the periphery. On the cold side, fluid is pumped from the periphery to the center, and leaves the heat exchanger as a single stream. Two experimental apparatuses were built and tested. In the first design, the body of the heat exchanger was made out of plexiglass and a peripheral plenum was used to collect or distribute the working fluid to the tree structure. The measurements showed that the use of a plenum generates undesirable volumetric flow asymmetries. These lessons led to a second design, which has two major improvements: (i) the heat exchanger core was made out of aluminum and (ii) individual ports (inlets/outlets) were used for each of the peripheral channels. The hydraulic results show a relation between the appearance of volumetric flow rate asymmetries and the bifurcation angles throughout the dendritic structure. Conceptually, their work represents a step toward the next generation of heat exchangers with maximum density.

Zimparov et al. [116] optimized the performance of several classes of simple flow systems consisting of T- and Y-shaped assemblies of ducts, channels and streams. In each case, the objective is to identify the geometric configuration that maximizes performance subject to several global constraints. Maximum thermodynamic performance is achieved by minimization of the entropy generated in the assemblies. The boundary conditions are fixed heat flow per unit length and uniform constant heat flux. The flow is assumed laminar and fully developed. Every geometrical detail of the optimized structure is deduced from the constructal law. Performance evaluation criterion is proposed for evaluation and comparison of the effectiveness of different tree-shaped design heat exchangers. This criterion takes into account and compares the entropy generated in the system with heat transfer performance achieved. Throughout these series of examples they showed that the optimized geometry has the effect of ‘partitioning’ optimally certain features of the system. Optimal partitioning or optimal allocation of constrained quantities is a byproduct of the optimization of flow geometry. It is encountered every time global performance is maximized: optimal allocation is another way of interpreting the special optimization of the flow arrangement, i.e., the optimal spreading of imperfection (irreversibilities). Zimparov et al. [117] extended the previous work by using boundary condition as the fixed temperature of the channel wall.

Wechsatol et al. [118] studied the effect of junction losses on the optimized geometry of tree-shaped flows. Several classes of flows are investigated systematically in a T-shaped construct with fixed internal and external size namely, laminar with non-negligible entrance and junction losses, and turbulent in tubes with smooth and rough walls. It is shown that in all cases junction losses have a sizeable effect on optimized geometry when $Sv^2 < 10$, where the svelteness Sv is a global property of the entire flow system which is defined as Sv=external length scale/internal length scale. The relationship between the global Sv and the slenderness of individual channels is discussed. The study showed that, in general, the duct slenderness decreases as the tree architecture becomes finer and more complex. They concluded that the miniaturization pushes flow architectures not only toward the smaller, finer and more complex, but also toward the domain in which junction losses must be taken into account in the optimization of geometry.

An analysis of the tree-like network construct method is presented for heat conduction optimization by Wu et al. [119]. The result confirms the expectation of constructal theory that the more complex the construct is, the less the thermal resistance and it is proved that the tree-like network construct method is faultless. Muzychka et al. [120] carried out analyses and comparisons of the multi-scale optimizations of constructal multi-scale design of compact micro-tube heat sinks and heat exchangers by using numerical calculation methods. It was shown that through the use of intersticial tubes, maximum heat transfer rates for arrays of circular tubes were increased, but did not surpass rates for arrays of parallel plates. Approximate results were obtained using Bejan's intersection of asymptotes method. These approximate solutions were compared with exact results using semi-analytical relationships for fluid friction and heat transfer in tubes. In general, through the use of multi-scale design techniques, greater performance of heat sink/heat exchanger core structures can be obtained versus conventional design approaches. The method is robust and may be applied to systems with different tube layouts from those considered, assuming that the distribution of tubes (or scales) is known. Finally, it was demonstrated that using the approximate solution for internal geometry, when combined with fundamental heat transfer theory for heat exchangers, provided excellent results for the heat transfer rate density.

Luo et al. [121] examined experimentally the effects of constructal distributors or collectors, built on a binary pattern of pores, on flow equidistribution in a multi-channel heat exchanger such as conventional pyramid distributors and a mini cross flow heat exchanger. The result showed a better thermal performance. Raja et al. [122] proposed the design and analysis of a multi-block heat exchanger by applying the concept of constructal theory. The heat exchanger works on the principle of developing laminar flow in each block carefully designed to avoid fully developed heat transfer coefficient. The additional thermal interaction is provided by the special design allowing heat transfer in ports as well as collecting and distributing channels. Numerical simulations were carried out for different values of heat capacity rate ratios on finned and unfinned constructal heat exchangers and four cross flow heat exchangers (two finned and two unfinned). In all the heat exchangers the heat transfer area is kept the same. To validate the numerical results, experiments were conducted keeping the heat transfer area and boundary conditions same as that of the numerical simulations. The results showed that the effectiveness of the constructal heat exchangers, both finned and unfinned, are higher by around 20% compared to that of the conventional cross flow heat exchangers under similar conditions. The experimental result confirms this enhancement and brings out the immense potential of this new type of heat exchanger.

The hydrodynamic performance of the network, composed of series of rough ducts, for both laminar and turbulent flow regimes is studied by Miguel [123]. Transient response of internal fluid pressure is also modeled and analyzed. The study is focused on the understanding of fluid flow and fluid pressure in a dendritic flow network. An approach is presented by combining hydrodynamics with a geometric description of the network. First, they examined steady and transient fluid flows. Finally, an approach was presented whereby sudden transient response of internal pressure variation is shown to be related with the geometry and mechanical properties of the network, as well as the fluid properties.

Kim et al. [124] showed numerically how the geometric configuration of the tubular flow structure controls the global performance of a cross-flow heat exchanger. The cold side is driven by natural convection and consists of a thermosyphon flowing in vertical tubes attached to two plenums. The hot side consists of hot gases that flow perpendicularly to the tubes, and heat the tubes by cross-flow forced convection. They showed numerically how the geometric configuration of the tubular flow structure controls the global performance of a cross-flow heat exchanger. The objective was to determine the number of tubes in cross flow such that the global heat transfer rate from the hot side to the warm side is maximum. Related to this objective was the maximization of the circulation rate in the vertical tubes of the thermosyphon. They found that the configuration for maximum convection is nearly the same as the configuration for maximum circulation rate. In other words, in the search of optimal flow

configuration it is sufficient to focus on the one global criterion, for example, the highest convection rate. They concluded that it is possible to determine the flow configuration by morphing the entire flow structure in pursuit of progressively higher global heat transfer rates between the hot side and cold side. This is in line with the method of design with constructal theory, and it is a clear departure from modern practice where the multi-tube flow configuration has a single tube size and tube spacing.

The structure of the regenerator consisting of an array of parallel plates in a thermo-acoustic engine is optimized for fixed cross-sectional area constraints based on constructal theory by Xuxian et al. [125]. The plate spacing or channel size and the plate number are taken as optimization objectives. The analytical formulas for plate spacing and the plate number of the thermo-acoustic stack are derived. The effects of some parameters on the optimal plate spacing or channel size and the optimal plate number are analyzed by numerical examples. The result presented provides theoretical guidelines for the design of the stack in thermo-acoustic engines.

Constructal analysis of tree-shaped microchannels for flow boiling in a disc-shaped body has been carried out to achieve an energy efficient design for chip cooling by Daguenet-Frick et al. [126]. They try to determine the best architecture that minimizes the thermal resistance for a given pressure drop and under several constraints. The expressions for the optimal plate spacing or channel size and the optimal plate number are obtained. The geometry is defined as n_0 channels touch the center and n_p channels touch the periphery. Three different complexities have been investigated: the radial flow pattern where $n_0=n_p$; the one pairing level flow pattern where $2n_0=n_p$; and the two pairing level flow pattern where $4n_0=n_p$. The fluid is R-134a evaporating at a temperature of 300 K. The fluid enters under a saturated state at the inlet and exits at the periphery. They concluded that increasing the number of channels results in decrease of the thermal resistance, whatever the complexity is. It is shown that the use of a radial structure with $2n_0$ central channels is more efficient than a one pairing level design with n_0 central channels. For low pumping power, the radial flow pattern presented the lowest thermal resistance. For medium pumping power, one pairing level design showed the lowest pumping power. For higher pumping power, the design with two pairing levels exhibits the best solution.

Flow boiling in constructal tree-shaped minichannel network is numerically investigated using a one-dimensional model, taking into consideration the minor losses at junctions by Zhang et al. [127]. The pumping power requirement, pressure drop, temperature uniformity and coefficient of performance of the constructal tree-shaped minichannel network are all evaluated and compared with those of the corresponding traditional serpentine channel, and the fluid stream undergoes a phase change from saturated liquid to saturated vapor.

Constructal entransy dissipation rate minimization of round tube heat exchanger cross-section was studied by Wei et al. [128]. The optimization results showed that the construct based on the minimum entransy dissipation rate could greatly reduce the average heat transfer temperature difference, and could improve its heat transfer performance. Wei et al. [128] discussions on entransy is based on concept introduced by Guo et al [37] and Guo et al [57] in order to publish as new concept of all the original advanced that have been published before them with the methods of entropy generation minimization, exergy analysis and the constructal law [1–6]. The concept in [128] is having lack of proper space in thermodynamics and in order to lend some credibility to “entransy”, the manuscript uses “constructal entransy”, which is based on the original works of [1–6]. The constructal entransy dissipation rate minimization, which for more credibility mixes of “entransy” with “entropy generation and thermal resistance minimization,” is not the original concept. The works of [37,56,57,128] used older concepts of constructal theory as new one by including entransy and renaming as new concept “constructal entransy”.

Some of the important aspects as provided by Herwig [40] are that even though entransy is not energy that it can be stored in a system. This means that it must be a state quantity. However, its characterization is twofold, entransy being an indication of the nature of energy (which is a state quantity), as well as that of the heat transfer ability (which is a process quantity). If entransy would be energy, the fact that it can be dissipated (i.e. transformed into another form of energy) would indicate its state quantity character. Since, however, entransy is not energy; dissipation of such a quantity should be defined along with introducing a new category of heat transfer relevant physical quantities. It has been shown that the dimension of entransy and entransy dissipation is JK, which contradicts the claim that it is energy quantity. In all the publications of entransy, there is mention of entropy and entropy generation as an alternative to entransy when irreversible heat transfer processes should be described and assessed. However, the entransy concept does not take into account the exergy and exergy losses in the processes.

The new approach of constructal theory has been employed to design shell and tube heat exchangers by Azad and Amidpour [129]. The results of design using the constructal theory are heat exchangers with in-series sections which are called as constructal shell and tube heat exchangers. In sections of the heat exchanger, optimal values of diameter and length are found by trade-off between operational and capital costs. The series structure of sections of constructal heat exchangers facilitates reparation, maintenance and deposit removal throughout their operation. The optimal configurations of constructal shell and tube heat exchangers are found using the genetic algorithm. The case study used in their paper for validation was obtained from one of the renowned reference texts on heat exchanger design represented more than 50% reduction in total cost compared to the usual method of design. Consequently, designing heat exchangers by the use of constructal theory is proposed as a useful method for designers, engineers and researchers.

Kim et al. [130] developed analytically the constructal design of steam generators with large number of tubes. Their study showed that the main features of a steam generator can be determined based on the method of constructal design. The generator is endowed with freedom to morph, and then is optimized by putting the right components in the right places. The number of steam tubes is sufficiently large, so that the steam generator may be modeled as continuous. The total volume of the assembly and the volume of the steam tubes are fixed. The geometry is free to vary in the search for maximum heat transfer density. The steam flow in the tubes is modeled in two ways namely single-phase and two-phase fully developed turbulent flow. The obtained results from the analysis are the location of the flow reversal (i.e. the demarcation between the tubes of the downcomer and those of the riser), the optimal spacing between adjacent tubes and the number of tubes for the downcomer and the riser.

Manjunath [131] used entropy generation minimization method to analyze different configurations of constructal heat exchangers designed by Bejan. Comparison of several tree-heat exchanger configurations such as trees covering uniformly a rectangular area, trees on a disk shaped area, and trees on a square-shaped area are studied based on heat transfer and pressure drop entropy generation formulations by varying number of pairing levels and initial length-to-diameter ratio for best performance. Second law analysis is able to choose the particular configuration of constructal heat exchanger for the required applications for lower entropy generation which leads to energy conservation.

Feng et al. [132] carried out constructal optimization for H-shaped multi-scale heat exchanger based on entransy theory.

Based on constructal theory, entropy generation minimization and second law efficiency equations are formulated for tree-shaped counter flow imbalanced heat exchanger for fully developed laminar and turbulent fluid flow by Manjunath and Kaushik [133]. Entropy generation number, rational efficiency and effectiveness behavior with respect to changes in number of pairing levels and different tube length-to-diameter ratios of constructal heat exchanger are analyzed analytically. Values of tube length-to-diameter ratio and number of pairing levels of constructal heat exchanger can be obtained for lower entropy generation number and higher second law efficiency which includes both irreversibilities due heat transfer and pressure drop. The analysis reveals the improvements in the performance of constructal heat exchanger compared to conventional single-tube dimensioned heat exchanger.

Comparison of a constructal heat exchanger and normal heat exchanger is analyzed by using second law analysis by Manjunath and Kaushik [134]. Analysis is carried out by considering the three irreversibilities due to heat transfer, pressure drop and production of the materials and the construction of the heat exchanger. The entropy generation minimization method is used to formulate the entropy generation number expressions of all three irreversibilities. Additionally, the thermo-economic aspect of the heat exchanger is considered to further analyze the economic differences between the constructal heat exchanger and normal heat exchanger. Graphical results are presented to investigate the influence of different parameters such as the number of pairing levels and initial length-to-diameter ratio on the behavior of the three entropy generation numbers along with effectiveness and *Ntu*. From the overall results, they found that there is an increase in the performance and a cost reduction in the constructal heat exchanger when compared to the normal heat exchanger. Constructal theory along with second law analysis will provide procedure for improving the performance of heat exchangers which leads in the conservation of energy.

4. Conclusions

This work first focuses on the importance of heat exchanger design from energy conservation point of view. The applications of heat exchanger are mentioned and performance evaluation based on heat transfer and thermodynamic analyses are provided. The importance of analysis of heat exchangers based on second law of thermodynamics is provided to study different types of losses occurring in the system. The constructal theory was introduced and its applications in heat exchangers are highlighted. The first law and second law of thermodynamics based analysis methodology of heat exchanger are provided briefly. Second law based entropy generation analysis is provided for different working fluids of heat exchangers namely, gas–gas, gas–liquid, liquid–liquid and two phase cases. Also, analysis based on entropy generation is provided for different types of heat exchangers namely, counter flow, parallel flow, cross flow mixed, cross flow unmixed etc. The advantages of second law analysis is that different types of irreversibilities occurring can be included in a single closed form of performance parameters expressions like entropy generation minimization number, exergetic or second law efficiency, etc. The approach for thermoeconomic analysis is mentioned which incorporates the costs associated with irreversibilities along with investment cost.

The critical analysis of literatures was carried out based on second law analysis such as, entropy generation as performance parameter, exergy analysis as performance parameter, second law analysis of heat exchanger considering production irreversibility, second law analysis of two-phase flow heat exchangers and constructal theory applied to heat exchangers. Also, brief introduction to constructal theory is provided with its broad applications.

Analysis of heat exchangers based on entropy generation as performance parameter has been carried out in terms of non-dimensional numbers in different ways. The entropy generation rate equation is divided by different terms like heat capacity rate, the ratio of heat transfer rate with reference or fluid inlet temperature, the ratio of heat transfer rate with fluid temperature difference, reference entropy generation rate and maximum entropy generation rate. Also, the analysis has been carried out in terms of different ways of defining entropy generation numbers like irreversibility distribution ratio, Bejan number and second law efficiency terms, enthalpy exchange irreversibility norm, quality of energy transformation and relative entropy generation.

Analysis of heat exchangers based on exergy as performance parameter has been carried out in terms of either second law efficiency such as rational efficiency or exergetic efficiency or non-dimensional numbers such as, merit function, non-dimensional exergy destruction number, exergy destruction number, heat transfer improvement number, specific irreversibility, dimensionless exergy loss rate, irreversible number, fractional exergy loss and exergy recovery index.

Second law analysis of heat exchanger considering production and manufacturing irreversibility has been carried out as an extension to the usual method by including the additional irreversibility term. Also, investigations have been carried out in terms of life cycle analysis by using the concept as life cycle irreversibility. The performance parameters used were exergetic life cycle analysis or minimization of life cycle irreversibility and cumulative exergy destruction due to material and manufacturing of heat exchanger.

Second law analysis of two phase heat exchangers has been carried out by investigators and applied to different types of heat exchangers based on applications. Either the method of exergy analysis or entropy generation was adopted by the investigators in their two phase applications. Also, to include the optimization aspect in their analysis, exergoeconomic or thermoeconomic method is added with the usual method. Thermoeconomic analysis which is the combination of cost of entropy generation or exergy destruction and capital cost of the equipment is carried out by several investigators to obtain better performance and cost effective solution of heat exchangers design.

The entropy generation analysis is preferred over exergy analysis, as entropy generation is a process quantity and does not need reference temperature. While, exergy analysis is an availability function consisting of enthalpy term, which is combination of state variable and process variable.

Also, it was discussed that entransy is a new physical quantity introduced as the theory of energy transfer in the form of heat with the aim of optimizing heat transfer which has got many thermodynamic difficulties in its description. The new quantity, which is defined, should be consistent with the theory as an extension to existing heat transfer processes. It should also provide the experimental evidences to prove itself as a new concept and approach. Further, in its present form, it has been shown that entransy is an unnecessary quantity because any of its applications can be reduced to more general entropy generation approach which is well established.

Constructal theory is used to optimize the performance of thermo-fluid flow systems by generating geometry and flow structure, and to explain natural self-organization and self-optimization. Constructal theory is applied in the design of heat exchangers by several investigators and widely accepted by researchers worldwide. Second law analysis along with constructal theory is applied to heat exchangers to improve the performance leading to energy conservation.

Bejan first described the constructal route to the conceptual design of a two-stream heat exchanger with maximal heat transfer rate per unit volume. This followed series of investigations using constructal theory broadly applied to heat exchangers in optimal tree-shaped networks for fluid flow in a disc-shaped body, dendritic fins optimization for a coaxial two-stream heat exchanger, constructal tree-shaped two-phase flow for cooling a surface, constructal multi-scale tree-shaped heat exchanger, constructal design of force convection cooled microchannel heat sinks and heat exchangers, thermodynamic optimization of tree-shaped flow geometries, constructal multi-scale design of compact micro-tube heat sinks and heat exchangers, constructal mini cross flow heat exchanger, thermal performance of a constructal multi-block heat exchanger, dendritic structures for laminar and turbulent fluid flow, constructal multi-tube configuration for natural and forced convection in cross-flow, flow boiling in constructal tree-shaped minichannel network, constructal theory based economic optimization of shell and tube heat exchanger, steam generator structure, second law analysis of unbalanced constructal heat exchanger and entropy generation and thermoeconomic analysis of constructal heat exchanger. Further work along this direction is in progress at global level.

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